

THUR 2-23-06

DIFFERENTIAL EQUATIONS

Ex) $\frac{dy}{dx} = x+1$; $y(2) = 3$; Find y .

INITIAL
CONDITION

$$y = \int (x+1) dx$$

$$y = \frac{1}{2}x^2 + x + C \quad \text{GENERAL SOLN}$$

$$(2, 3)$$

$$3 = \frac{1}{2}(2)^2 + 2 + C$$

$$3 = 2 + 2 + C$$

$$\underline{\underline{-1 = C}}$$

$$\therefore y = \frac{1}{2}x^2 + x - 1$$

CHECK?

GRAPHICAL SOLUTION

"SLOPE FIELD" (ON AP EXAM)

①

```

F1- Tools  F2- Zoom  F3- Edit  F4- ✓  F5- All  F6- Style  F7- ?
+PLOTS
  t0=2.
  y1'=t+1
  y1=3.
  y2'=
  y2=
  y3'=
  y3=
y2'(t)=
MAIN      RAD AUTO  DE
    
```

(2, 3)

③

```

F1- Tools  F2- Zoom  F3- Trace  F4- ReGraph  F5- Math  F6- Draw  F7- Pen  F8- IC
tc:2.
xc:2.
yc:3.
USE ←→ ON TYPE + [ESC]=CANCEL
    
```

TRACE

②

```

F1- Tools  F2- Zoom
t0=2.
tmax=10.
tstep=.1
tplot=2.
xmin=-5.
xmax=5.
xscl=1.
ymin=-10.
ymax=10.
yscl=1.
MAIN      RAD AUTO  DE
    
```

④

```

F1- Tools  F2- Zoom  F3- Trace  F4- ReGraph  F5- Math  F6- Draw  F7- Pen  F8- IC
tc:-1.
xc:-1.
yc:-1.5
    
```

✓ VERTEX

$$y = \frac{1}{2}x^2 + x - 1 \quad (-1, -1\frac{1}{2})$$

Given: $\frac{dy}{dx} = x^2$; $y(0) = 4$; $y = ?$

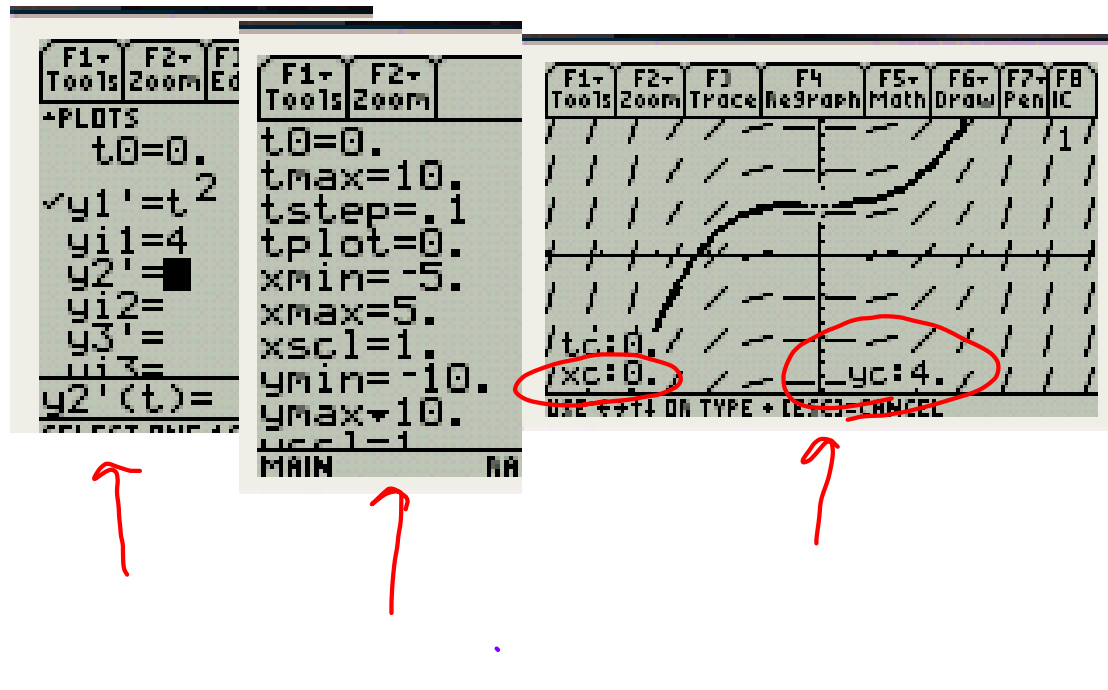
Soln: $y = \int x^2 dx = \frac{1}{3}x^3 + C$

$4 = \frac{1}{3}(0)^3 + C \rightarrow C = 4$

$y = \frac{1}{3}x^3 + 4$ ✓

SOLUTION IS
A FUNCTION

GRAPHICAL SOLN:



Application Problem

Suppose \$1000 is invested in an account that pays 7.2% interest compounded continuously. Find a formula for the amount of money in the account at any time t (in years).

SOLN: LET $t=0$ WHEN THE INITIAL \$1000 IS INVESTED. LET y = AMOUNT OF INVESTMENT AT ANY TIME t ; $y(t) = \dots$

MODEL: $\frac{dy}{dt} = .072y$; $y(0) = 1000$

Hmmm... $y' = .072y$

WHAT FUNCTION'S DERIVATIVE IS A CONSTANT MULTIPLIER OF ITSELF?

$y = x^2$ $y' = 2x$ \rightarrow $y' = k \cdot x^2$
NO

$y = e^{3x} \rightarrow y' = 3 \cdot e^{3x}$

$y(t) = C \cdot e^{k \cdot t}$

C, k CONSTANTS
WE KNOW 1 OF THEM.

$$y(t) = C \cdot e^{k \cdot t}$$

C, k CONSTANTS
WE KNOW 1 OF
THEM.

$$k = .072$$

$$y(t) = C \cdot e^{.072t}$$

$$\rightarrow y'(t) = \underbrace{.072 \cdot C}_{\text{CONSTANT}} \cdot \underbrace{e^{.072t}}_y$$

$y' = \text{CONSTANT} \cdot y$

$$y = A \cdot e^{.072t}$$

$t=0, y=1000$
 $1000 = A \cdot e^0$

$$\underline{.072C = A}$$

$$y(t) = 1000 \cdot e^{.072t}$$

$$y' = .072y$$

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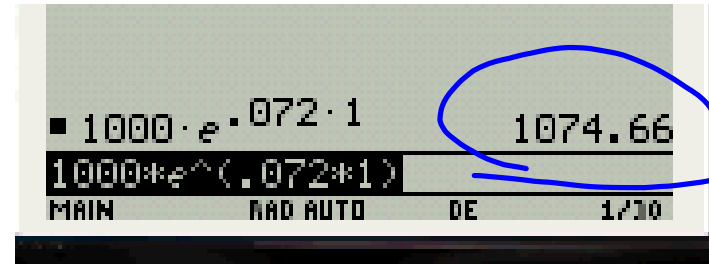
$$\underline{y = P \cdot e^{rt}}$$

$$y(x) = 1000 \cdot e^{.072x}$$

AT END OF 1 YR How much money?

$$y(x=1) = 1000 \cdot e^{.072(1)}$$

$$= \underline{\underline{1074.66}}$$

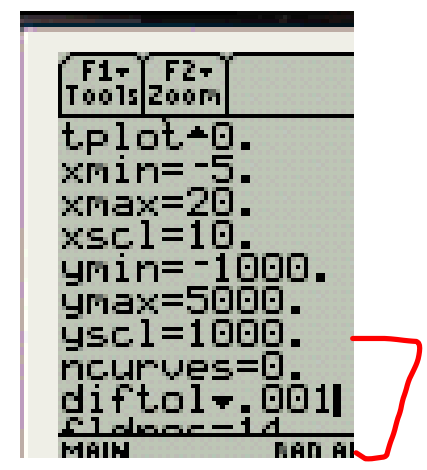
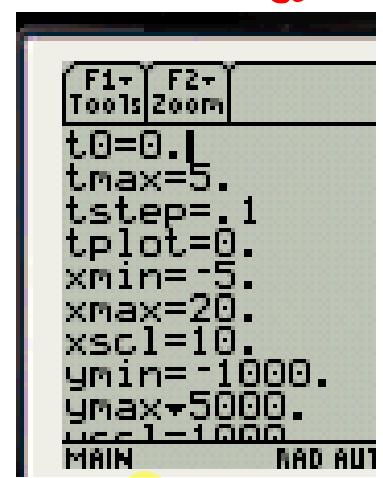
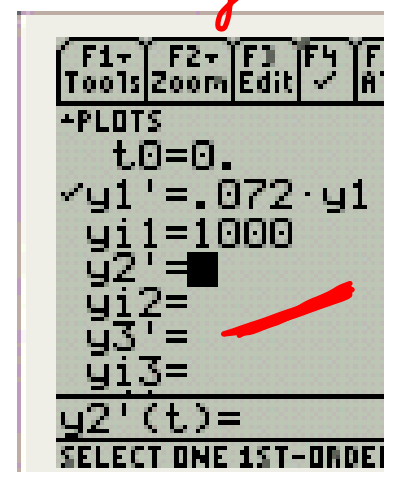


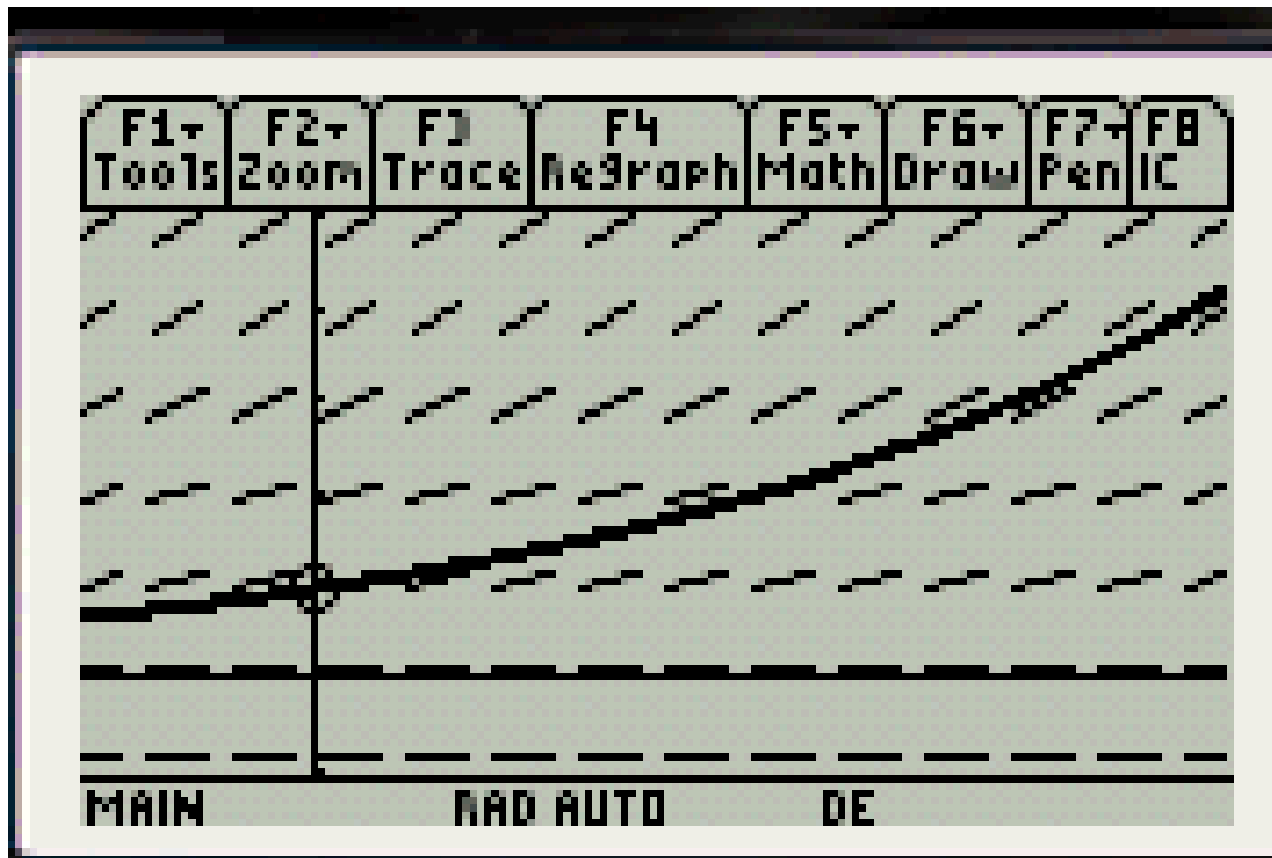
DIFF EQ:

$$y' = .072 \cdot y$$

y =

WINDOW





ANSWERS TO EVENS

● (6) $\ln |x+3| + C$

● (10) $x^{\frac{4}{3}} + C$ OR $x \cdot \sqrt[3]{x} + C$

● $\int \cos 2x \, dx = \frac{1}{2} \sin 2x + C$
BY TRIAL 'n' SUCCESS

● SEE p. 307 FOR TRIG. DIFFERENTIATION

● O.T.L. p. 312-4 14, 16, 18, 27, 28,
31, 34, 59, 43

● $\int 4 \sin 2x \, dx = ?$ (NO G.C.)