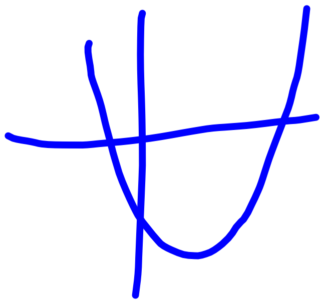
THUR 03-09-06

AP YELLOW 27 C

27 $f'(x)$ IS INC

$f''(x)$ IS POS



f IS INC → f' IS POS

6.5 ONE COMMON WAY TO MODEL POPULATION IS WITH A DIFFERENTIABLE FUNCTION P GROWING AT A RATE PROPORTIONAL TO THE SIZE OF THE POPULATION.

WRITE THIS AS AN EQUATION.

$$\frac{dP}{dt} = k \cdot P$$

SOLVE THE DIFF. EQ.

$$\frac{dP}{dt} = k \cdot P$$

*SEPARATE THE VARS.

$$\frac{dP}{P} = k \cdot dt$$

$$\int \frac{1}{P} \cdot dP = k \int dt$$

$$\ln|P| + C_1 = kt + C_2$$

$$\ln P = kt + C_3$$

$$C_3 = C_2 - C_1$$

$$\log_e P = kt + C_3$$

$$e^{kt + C_3} = P$$

$$e^{kt} \cdot e^{C_3} = P$$

$$A = e^{C_3}$$

$$P = A \cdot e^{kt}$$

$$P(t=0) = P_0$$

$$P = A \cdot e^0 = P_0 = A$$

$$\boxed{P(t) = P_0 \cdot e^{kt}}$$

*

P. 348 (20) DIFF. EQ.: $\frac{dP}{dt} = k \cdot P$

5000 TO 10,000

i) $\frac{dP}{dt} = \frac{1}{4} \cdot P$

$P = P_0 \cdot e^{\frac{1}{4}t}$

$\frac{10000}{5000} = \frac{5000 \cdot e^{\frac{1}{4}t}}{5000}$

$2 = e^{\frac{1}{4}t}$

$\ln 2 = \ln e^{\frac{1}{4}t}$

$\ln 2 = \frac{1}{4}t \cdot \cancel{\ln e}^{1}$

$\underline{4 \ln 2 = t_1}$

10,000 TO 25,000

$\frac{dP}{dt} = \frac{1}{12} P \; ; \; k = \frac{1}{12}$

$P = P_0 \cdot e^{\frac{1}{12}t}$

$\frac{25{,}000}{10{,}000} = \frac{10{,}000 \cdot e^{\frac{1}{12}t}}{10{,}000}$

$2.5 = e^{\frac{1}{12}t}$

$\ln 2.5 = \ln e^{\frac{1}{12}t}$

$\ln 2.5 = \frac{1}{12}t \cdot \cancel{\ln e}^{1}$

$\underline{12 \cdot \ln 2.5 = t_2}$

TOTAL TIME $= 4\ln 2 + 12 \cdot \ln 2.5$ (EXACT)

$\doteq 13.768$ yrs.

O.T.L. · P. 338 2, 8, 12, 15

· P. 346 QUICK REVIEW 1-8
(USE CAS)

· P. 347-9 1a, b, 5, 16, 21

AP p. 29-32 87-90