WED 5-10-06

BACK TO VOLUMES OF REVOLUTION

DONE SO FAR:

DISC METHOD

INTRODUCING . . .

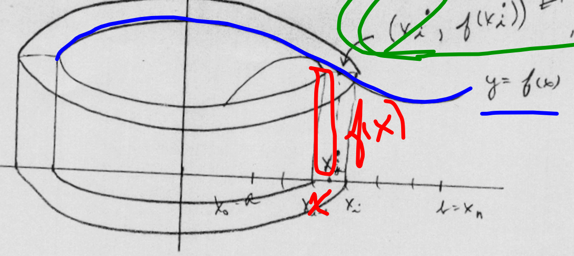
THE SHELL METHOD!

VOLUMES : SHELL METHOD

DERIVATION

ROTATE $y = f(x)$ ABOUT THE y-AXIS.

$\pi \cdot r^2 \cdot h$



VOLUME IS APPROXIMATED BY SUMMING VOLUMES OF "OUTSIDE" CYLINDERS AND "INSIDE" CYLINDERS. — "SHOTGUN SHELLS"

OUTSIDE INSIDE

$$V \doteq \sum_{i=1}^{n} \left[\pi \cdot x_i^2 \cdot f(x_i^*) - \pi \cdot x_{i-1}^2 \cdot f(x_i^*) \right]$$

$$= \sum_{i=1}^{n} \pi \cdot f(x_i^*) \left[x_i^2 - x_{i-1}^2 \right]$$

ACTUAL VOLUME IS: $V = \lim \sum_{i=1}^{n} \pi \cdot f(x_i^*) \left[x_i^2 - x_{i-1}^2 \right]$

$$= \lim \sum_{i=1}^{n} \pi \cdot f(x_i^*) (x_i - x_{i-1}) (x_i + x_{i-1})$$

BUT THIS IS Δx_i

$$= \lim \sum_{i=1}^{n} \pi \cdot f(x_i^*) (x_i + x_{i-1}) (\Delta x_i)$$

$\boxed{\text{* KEY}}$ USE THE MIDPOINT FOR), THAT IS $x_i + x_{i-1} = \frac{(x_i + x_{i-1})}{2} \cdot 2$

$$= \lim \sum_{i=1}^{n} \pi \cdot f(x_i^*) \left(\frac{x_i + x_{i-1}}{2}\right) (2) (\Delta x_i)$$

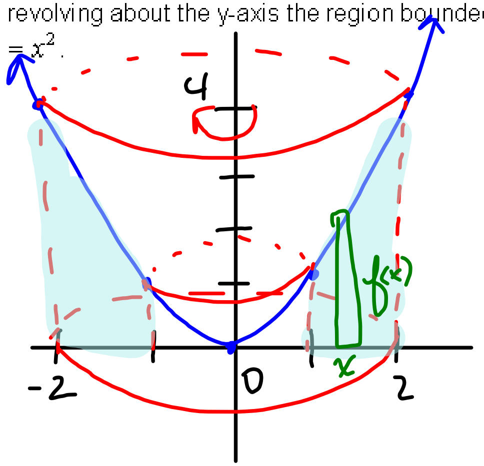
$$= \lim \sum_{i}^{n} 2\pi \cdot f(x_i^*) (x_i^*) (\Delta x_i)$$

TAKE THE LIMIT

$$\boxed{V = \int_a^b 2\pi \cdot f(x) \cdot x \cdot dx}$$

AP CALCULUS SECTION 7.3 GIFT 3 SHELL METHOD

1. Find the volume of the solid generated by revolving about the y-axis the region bounded on [1, 2] by the x-axis and the graph of $f(x) = x^2$.



$$V = \int_{x=a}^{x=b} 2\pi \cdot x \cdot f(x) \cdot dx$$

$$V = \int_1^2 2\pi \cdot x \cdot x^2 \, dx$$

$$V = 2\pi \int_1^2 x^3 \, dx$$

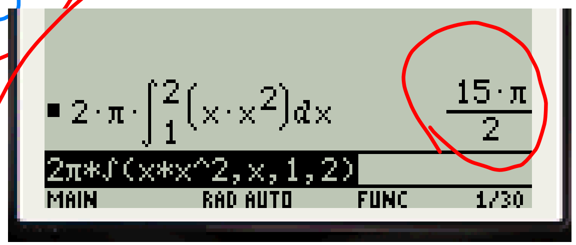
$$V = 2\pi \left[\frac{1}{4}x^4\right]_{x=1}^{x=2}$$

$$V = 2\pi \left[\frac{1}{4}(2)^4 - \frac{1}{4}(1)^4\right]$$

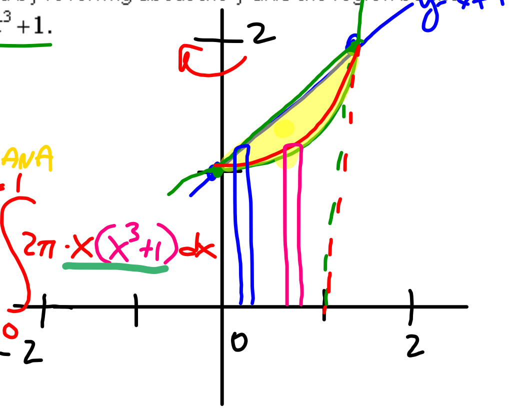
$$V = 2\pi \left[\frac{16}{4} - \frac{1}{4}\right]$$

$$V = 2\pi \cdot \frac{15}{4}$$

$$V = \frac{15\pi}{2} \text{ CU. UNITS}$$



2. Find the volume of the solid generated by revolving about the y-axis the region bounded by the graphs of $y = x+1$ and $y = x^3+1$.



$$V_{GRN} - V_{RED} = V_{BANANA}$$

$$V_B = \int_{x=0}^{x=1} 2\pi x(x+1)\,dx - \int_{x=0}^{x=1} 2\pi \cdot x(x^3+1)\,dx$$

$$V_B = 2\pi \int_{x=0}^{x=1} \left(x^2+x-(x^4+x)\right) dx$$

$$V_B = 2\pi \int_0^1 (x^2-x^4)\,dx$$

$$V_B = 2\pi \left[\frac{1}{3}x^3 - \frac{1}{5}x^5\right]_{x=0}^{x=1}$$

$$= 2\pi\left(\frac{1}{3}\cdot 1 - \frac{1}{5}\cdot 1 - 0\right)$$

$$= 2\pi\left(\frac{1}{3} - \frac{1}{5}\right)$$

$$= 2\pi\left(\frac{5}{15} - \frac{3}{15}\right)$$

$$= 2\pi \cdot \frac{2}{15}$$

$$= \frac{4\pi}{15} \text{ CU. UNITS}$$

O.T.L.

7.3 GIFT 3 SHELL METHOD

3-7 ONLY

HELP! ANSWERS (SOME OF THEM ARE EVEN CORRECT):

3 $\frac{\pi}{2}$ 4 $\frac{62\pi}{5}$ 5 $\frac{5\pi}{14}$ 6 5π

7 $\frac{52\pi}{15}$