TUES 11-15-05

P. 156 (23) 4

(25) 7

P. 162 (9) 1

(26) $y' = \frac{1}{y+1}$

$y'' = \frac{-1}{(y+1)^3}$

(23) $x^2 + y^2 = 1$

$2x + 2y \cdot y' = 0$

$\frac{2y \cdot y'}{2y} = \frac{-2x}{2y}$

$y' = -\frac{x}{y}$

$y' = -\frac{x}{y}$ Find y''

$$y'' = \frac{y \cdot -1 - y' \cdot -x}{y^2}$$

$$y'' = \frac{-y + x \cdot y'}{y^2}$$

$$y'' = \frac{-y + x \cdot \frac{-x}{y}}{y^2}$$

$$y'' = \frac{-y - \frac{x^2}{y}}{y^2} \cdot \frac{y}{y}$$

$$y'' = \frac{-y^2 - x^2}{y^3} \quad \text{or} \quad -\frac{x^2+y^2}{y^3}$$

$$= \frac{-1}{y^3}$$

· EXPLORATION #1

IF THE SLOPE OF THE LINE TANGENT TO $f(x)$ AT (a, b) IS $\frac{c}{d}$, WHAT IS THE SLOPE OF THE LINE TANGENT TO $f^{-1}(x)$ AT SOME POINT?

WHAT IS THE POINT? (b, a)

WHAT IS THE SLOPE? $\frac{d}{c}$

SUMMARIZE ABOUT INVERSE FUNCTIONS

- "JUST" SWITCH x & y
- GRAPHS OF f & f^{-1} ARE SYMMETRIC ABOUT $y = x$
- $f(f^{-1}(a)) = a = f^{-1}(f(a))$

DERIVE: $\frac{d(\cos^{-1}x)}{dx} = ?$

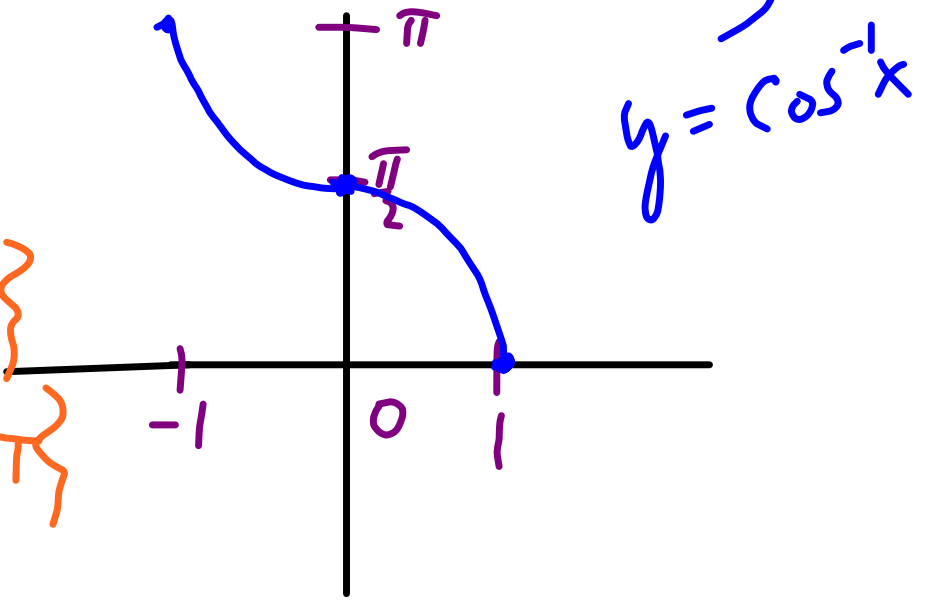
(IN TERMS OF x)

BACKGROUND:

GRAPH:

$D = \{x : x \in \mathbb{R}, -1 \le x \le 1\}$

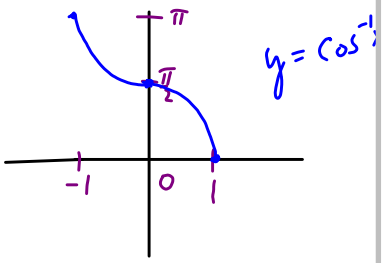
$R = \{y : y \in \mathbb{R}, 0 \le y \le \pi\}$



$\frac{d(\cos^{-1}x)}{dx} = ?$

$y = \cos^{-1}x$

REWRITE: $x = \cos y$



$y = \cos^{-1}x$

$\frac{d(x)}{dx} = \frac{d(\cos y)}{dx}$

$1 = -\sin y \cdot y'$

$\frac{-1}{\sin y} = y'$

$x = \cos y$

$x^2 = \cos^2 y$

$x^2 = 1 - \sin^2 y$

SOLVE FOR $\sin y$

$\sin^2 y = 1 - x^2$

$\sin y = \pm\sqrt{1-x^2}$

ONLY + BC $y \in$ I or II

CAN'T BE BOTH +/-

$y' = \frac{-1}{\pm\sqrt{1-x^2}}$

$y' = \frac{-1}{\sqrt{1-x^2}}$

GENERALIZE:

$y = \cos^{-1} u$, u IS A FUNCTION OF x

$$y' = \frac{-1}{\sqrt{1-u^2}} \cdot u'$$

HEY!

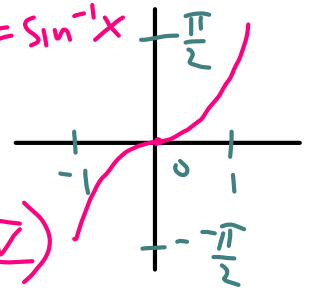
$\frac{d(\sin^{-1}x)}{dx} = ?$ IN TERMS OF x

BACKGROUND

D: $-1 \le x \le 1$

R: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(QUADS: I, IV)



$y = \sin^{-1}x$

$x = \sin y$

$1 = \cos y \cdot y'$

$\frac{1}{\cos y} = y'$

$x^2 = \sin^2 y$

$x^2 = 1 - \cos^2 y$

$\cos^2 y = 1 - x^2$

$\cos y = \pm\sqrt{1-x^2}$

$\cos y = +\sqrt{1-x^2}$

cos is pos in I & IV!

$y' = \frac{1}{\sqrt{1-x^2}}$

$\frac{d(\sin^{-1}u)}{dx}$

$= \frac{1}{\sqrt{1-u^2}} \cdot u'$

Ex) $y = \sin^{-1}(x^2 - 2x)$; $y' = ?$

$$y' = \frac{1}{\sqrt{1-(x^2-2x)^2}} \cdot (2x-2)$$

DERIVE USED:

$$\frac{d(\tan^{-1} u)}{dx} = \frac{1}{u^2+1} \cdot u'$$

O.T.L.
- FINISH EX P. 1
- BE ABLE TO DERIVE THE FORMULAS WE DID IN CLASS
- MEMORIZE THESE FORMULAS
- P.162 1-13 (ODD)