MON 11-28-05

TEST RESULTS FROM 3.7-3.9
HAVE BEEN POOR SOME YEARS.
WHY?

- STUDENTS DID NOT MEMORIZE ACCURATELY ALL THE DERIVATIVE FORMULAS
- THE "CHAIN RULE" WAS IGNORED – "STUFF"
- STUDENTS HAVE BECOME TOO DEPENDENT UPON CHECKING ANSWERS IN THE "BACK OF THE BOOK"
- "CARELESS" ERRORS ARE COSTLY – CAUSED BY NOT DOING ENOUGH CONSCIENTIOUS PRACTICE PROBLEMS
- DO PROBLEMS, WRITE FORMULAS, DON'T JUST "LOOK OVER"

AP Calculus Gift 3.9 In Class: How Fast Does a Flu Spread?

The spread of a flu in a certain school is modeled by the equation: $P(t) = \dfrac{100}{1+e^{3-t}}$ ' where

P(t) is the total number of students infected t days after the flu was first noticed. Many of them may already be well again at time t.

a) Estimate the initial number of students infected with the flu.
b) How fast is the flu spreading after 1 day? After 2 days?
c) When will the flu spread at its maximum rate? What is this rate?

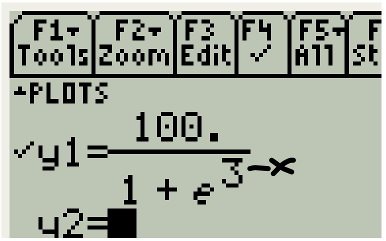
a) Estimate the initial number of students infected with the flu.

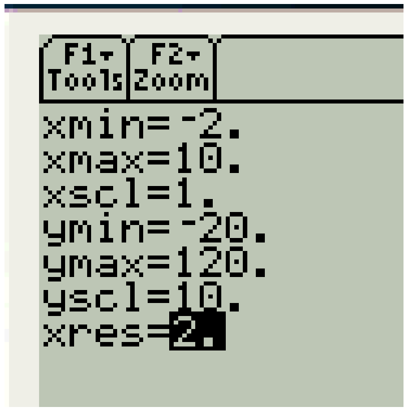
$$P(t) = \frac{100}{1+e^{3-t}}$$

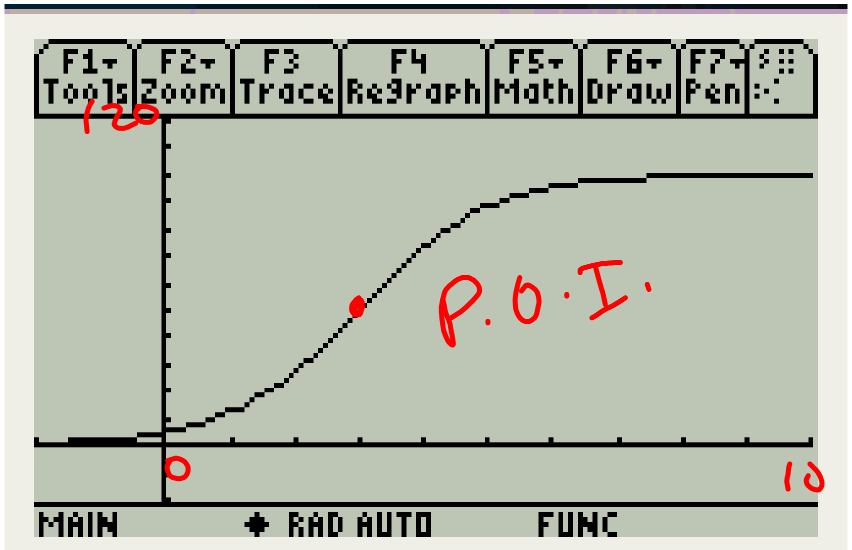
$$P(t=0) = \frac{100}{1+e^{3}}$$

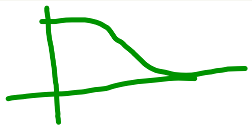
$\doteq 5$ STUDENTS

"LOGISTIC CURVE"









b) How fast is the flu spreading after 1 day? After 2 days?

$$P(t) = \frac{100}{1+e^{3-t}}$$

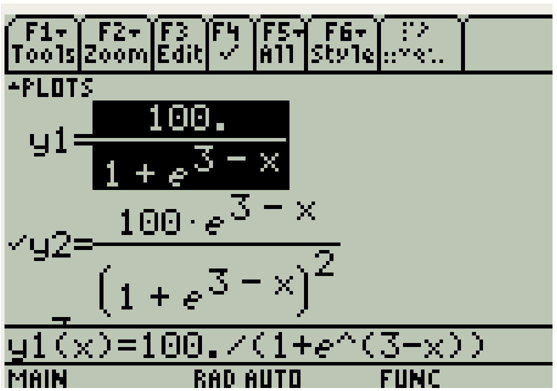
$P'(t) = ?$ IS A RATE OF CHANGE

REWRITE

$$P(t) = 100\left(1+e^{3-t}\right)^{-1}$$

$$P'(t) = -1 \cdot 100 \cdot \left(1+e^{3-t}\right)^{-2} \cdot e^{3-t} \cdot -1$$

$$\boxed{P'(t) = \frac{100e^{3-t}}{\left(1+e^{3-t}\right)^2}}$$



t	$y'(t)$
0	
1	NEXT
2	PAGE
3	
4	
5	

F1 Tools | F2 Zoom | F3 Edit | F4 ✓ | F5 All | F6 Style

PLOTS

$$y1=\frac{100.}{1+e^{3-x}}$$

$$\checkmark y2=\frac{100\cdot e^{3-x}}{(1+e^{3-x})^2} \leftarrow y'(t)$$

y1(x)=100./(1+e^^(3-x))

MAIN RAD AUTO FUNC

F1 | F2 | F3 | F4 | F5 | F6

TABLE SETUP

tblStart........ 0.

Δtbl........ 1.

Graph <-> Table OFF →

Independent....... AUTO →

Enter=SAVE ESC=CANCEL

y1(x)=100./(1+e^^(3-x))

TYPE + [ENTER]=OK AND [ESC]=CANCEL

F1 Tools | F2 Setup

x (t)	y2 ($y'(t)$)
0.	4.5177
1.	10.499
2.	19.661
3.	25.
4.	19.661

x=0.

MAIN RAD AUTO

DAY 1

← 10.5

STUDENTS INFECTED

DAY

5.	10.499
6.	4.5177
7.	1.7663
8.	.66481
9.	.24665

x=9.

MAIN RAD AUTO

DAY2

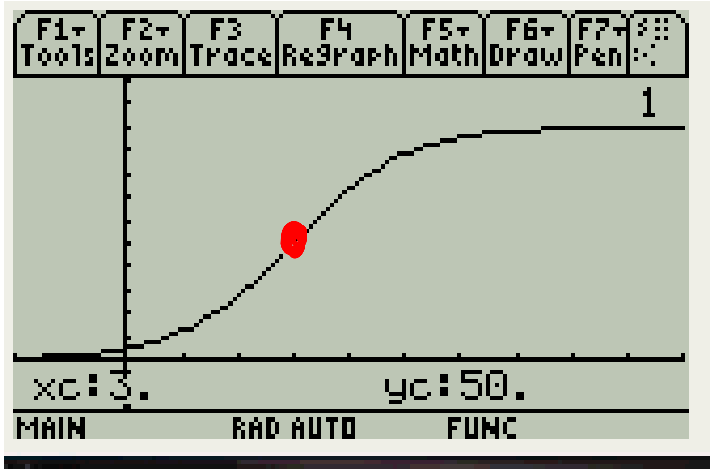
c) When will the flu spread at its maximum rate? What is this rate?

$$P(t) = \frac{100}{1+e^{3-t}}$$

25 $\frac{\text{STUDENTS INFECTED}}{\text{DAY}}$

DAY 3

SEE PREVIOUS TABLE



P.O.I.!

(19) $y = x^{\ln x}$

log. diff.

$\ln y = \ln x^{\ln x}$

$\ln y = \ln x \cdot \ln x = (\ln x)^2$

$\frac{1}{y} \cdot y' = 2(\ln x) \cdot \frac{1}{x}$

$y' = 2 \cdot \ln x \cdot \frac{1}{x} \cdot y$

$$y' = \frac{2 \cdot \ln x \cdot x^{\ln x}}{x}$$

39

43

45 NEXT PAGES

47

49

39) $y = \log_{10} e^x$

NOTE: $u = e^x$, $a = 10$

$$\frac{d(\log_a u)}{dx} = \frac{1}{u \cdot \ln a} \cdot \frac{du}{dx}$$

$$y' = \frac{1}{\cancel{e^x} \cdot \ln 10} \cdot \cancel{e^x}$$

$$y' = \frac{1}{\ln 10}$$

(43) $y = (\sin x)^x$

$\ln y = \ln\left((\sin x)^x\right)$

$\ln y = x \cdot \ln(\sin x)$

PROD. RULE

TAKE THE DERIV. OF EACH SIDE:

$\frac{1}{y} \cdot y' = x \cdot \frac{1}{\sin x} \cdot \cos x + 1 \cdot \ln(\sin x)$

"STUFF"

$y' = y \cdot \left[x \cdot \cot x + \ln(\sin x)\right]$

$$y' = (\sin x)^x \left[x \cdot \cot x + \ln(\sin x)\right]$$

(45) $y = \left(\dfrac{(x-3)^4(x^2+1)}{(2x+5)^3} \right)^{\frac{1}{5}}$

$$\ln y = \frac{1}{5} \cdot \ln \left(\frac{(x-3)^4(x^2+1)}{(2x+5)^3} \right)$$

*USE PROPERTIES OF ln:

$$\ln y = \frac{1}{5}\left(\ln(x-3)^4 + \ln(x^2+1) - \ln(2x+5)^3 \right)$$

$$\ln y = \frac{1}{5}\left(4 \cdot \ln(x-3) + \ln(x^2+1) - 3\ln(2x+5) \right)$$

$$\ln y = \frac{4}{5} \cdot \ln(x-3) + \frac{1}{5}\ln(x^2+1) - \frac{3}{5}\ln(2x+5)$$

TAKE THE DERIVATIVE OF EACH TERM:

$$\frac{1}{y} \cdot y' = \frac{4}{5} \cdot \frac{1}{x-3} \cdot 1 + \frac{1}{5} \cdot \frac{1}{x^2+1} \cdot 2x - \frac{3}{5} \cdot \frac{1}{2x+5} \cdot 2$$

NEXT PAGE

(45) 2^{nd} PAGE

$$\frac{1}{y}\cdot y' = \frac{4}{5}\cdot\frac{1}{x-3}\cdot 1 + \frac{1}{5}\cdot\frac{1}{x^2+1}\cdot 2x - \frac{3}{5}\cdot\frac{1}{2x+5}\cdot 2$$

SIMPLIFY:

$$\frac{1}{y}\cdot y' = \frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x+5)}$$

MULT. BY y:

$$y' = \left[\frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x+5)}\right]\cdot y$$

REPLACE y:

$$y' = \left[\frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x+5)}\right]\cdot \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}$$

WHEW!

47 $A(t) = 20\left(\frac{1}{2}\right)^{\frac{t}{140}}$

REWRITE:

$A(t) = 20(2)^{-\frac{t}{140}}$

RULE:

$\frac{d\,a^u}{dx} = a^u \cdot \ln a \cdot \frac{du}{dx}$

$A'(t) = \cancel{20} \cdot 2^{-\frac{t}{140}} \cdot \ln 2 \cdot -\frac{1}{\cancel{140}_{7}}$

$A'(t) = \frac{-(\ln 2) \cdot 2^{-\frac{t}{140}}}{7}$

$A'(t=2) = \frac{-(\ln 2) \cdot 2^{-\frac{2}{140}}}{7}$

$\doteq -0.098 \frac{\text{grams}}{\text{day}}$

MEANS A DECAY

(49) $f(x) = 2^x$

a) $f'(x) = 2^x \cdot \ln 2$

$f'(x=0) = 2^0 \cdot \ln 2$

$= \underline{\underline{\ln 2}}$

b) RECALL: $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

HERE $a = 0$; $f(x) = 2^x$

$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

$f'(0) = \lim_{h \to 0} \frac{2^h - 1}{h}$

c) THE ANSWERS IN a & b ARE EQUAL:

$$\underline{\underline{\lim_{h \to 0} \frac{2^h - 1}{h} = \ln 2}}$$

d) SIMILARLY: $\underline{\underline{\lim_{h \to 0} \frac{7^h - 1}{h} = 7}}$

Title: Nov 28 - 1:41 PM (14 of 15)

GOOD LUCK ON TOMORROW'S

TEST: 3.7 - 3.9