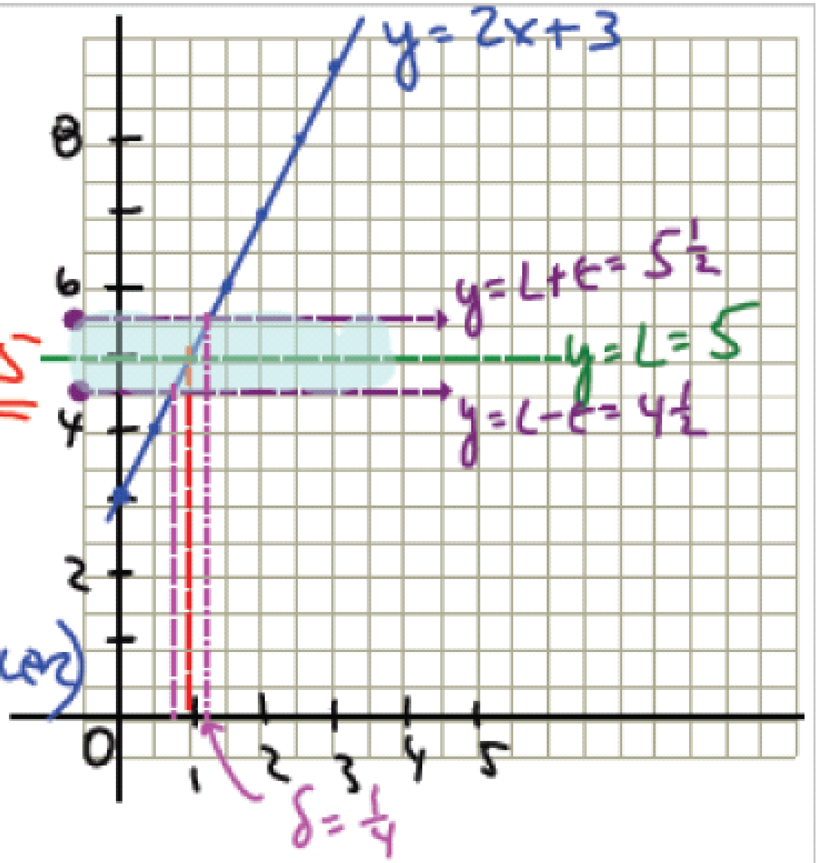
MON 5-12-08

KNOW THE LIMIT DEFINITION!

P.72 L-2

(12)

a)



b) $\lim_{x \to 1} (2x+3) = \underline{\underline{5}}$

c) $\underline{\underline{\delta = \frac{1}{4}}}$ (OR SMALLER)

12d) PROVE: $\lim_{x \to 1} (2x+3) = 5$.

<u>proof</u>: LET $\epsilon > 0$ BE GIVEN TO ME.

I MUST FIND $\delta > 0$ SUCH THAT:

$|2x+3-5| < \epsilon$ WHENEVER $0 < |x-1| < \delta$

$|2x-2| < \epsilon$

$|2(x-1)| < \epsilon$

$2|x-1| < \epsilon$

$|x-1| < \frac{\epsilon}{2}$

$\therefore$ CHOOSE $\underline{\underline{\delta = \frac{\epsilon}{2}}}$ (OR SMALLER)

(13) PROVE: $\lim_{x \to -3} (5x+9) = -6$

proof: LET $\epsilon > 0$ BE GIVEN TO ME.

I MUST FIND $\delta > 0$ SUCH THAT:

$|5x+9-(-6)| < \epsilon$ WHENEVER $0 < |x-(-3)| < \delta$

$|5x+15| < \epsilon$ $\qquad$ a $0 < |x+3| < \delta$

$|5(x+3)| < \epsilon$

$5|x+3| < \epsilon$

$|x+3| < \frac{\epsilon}{5}$

$\therefore$ CHOOSE $\delta = \frac{\epsilon}{5}$ (OR SMALLER)

(14) PROVE: $\lim_{x \to 4} \left(\frac{1}{2}x - 3 \right) = -1$

proof: LET $\epsilon > 0$ BE GIVEN TO ME.

I MUST FIND $\delta > 0$ SUCH THAT:

$\left| \frac{1}{2}x - 3 - (-1) \right| < \epsilon$ WHENEVER $0 < |x-4| < \delta$

$\left| \frac{1}{2}x - 2 \right| < \epsilon$

$\left| \frac{1}{2}(x-4) \right| < \epsilon$

$\frac{1}{2}|x-4| < \epsilon$

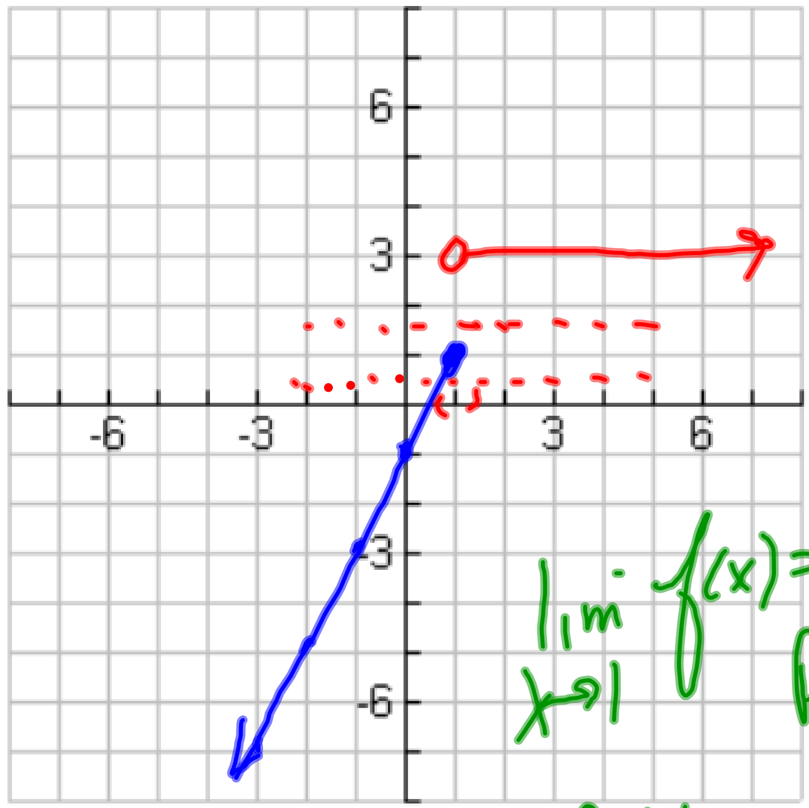
$|x-4| < 2\epsilon$

$\therefore$ CHOOSE $\delta = 2\epsilon$ (OR SMALLER)

"COUNTER EXAMPLE"

PIECE-WISE FUNCTION

$$f(x) = \begin{cases} 2x-1 & x \le 1 \\ 3 & x > 1 \end{cases}$$



$\lim_{x \to 1} f(x) = ?$

D.N.E.

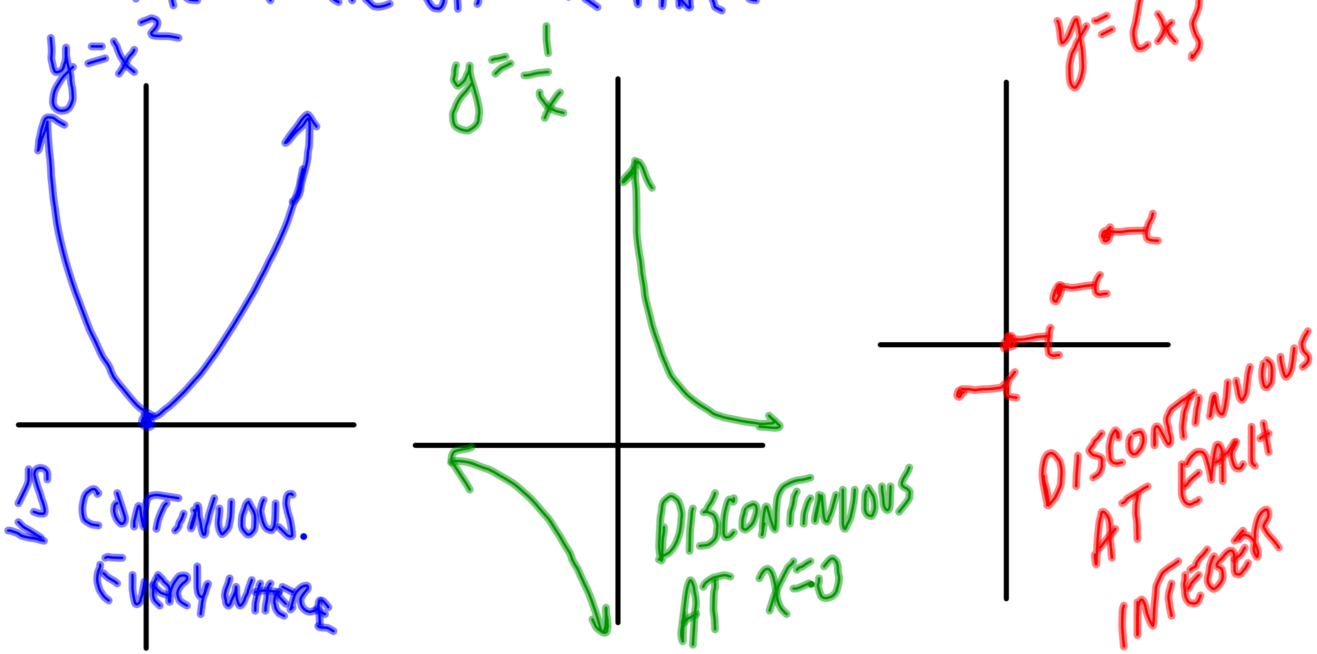
L.H.L. 1

R.H.L. 3

CONTINUITY

INTUITIVE

A FUNCTION IS SAID TO BE CONTINUOUS IF YOU CAN DRAW ITS GRAPH WITHOUT LIFTING THE PENCIL OFF THE PAPER.



FORMAL DEFINITION

A FUNCTION, f, IS SAID TO BE CONTINUOUS AT $x=a$ IF 3 CRITERIA ARE MET:

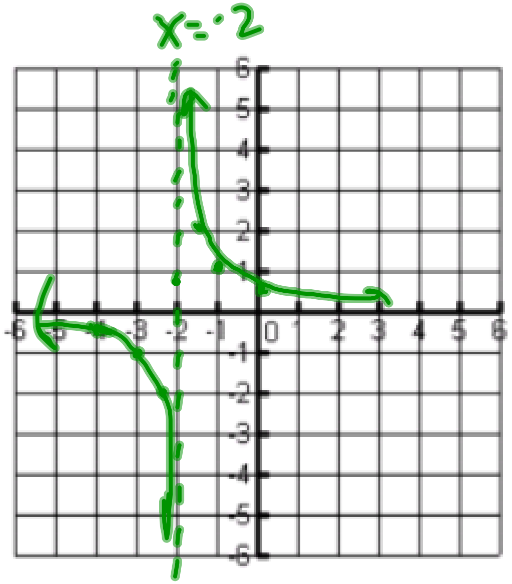
1) $f(a)$ EXISTS

AND 2) $\lim_{x \to a} f(x)$ EXISTS

AND 3) $\lim_{x \to a} f(x) = f(a)$

Continuity "In Class" Gift

Ex. 1 $f(x) = \dfrac{1}{x+2}$. Is f continuous at x = – 2? Explain.



NO!

$f(-2)$ D.N.E.

- OR -

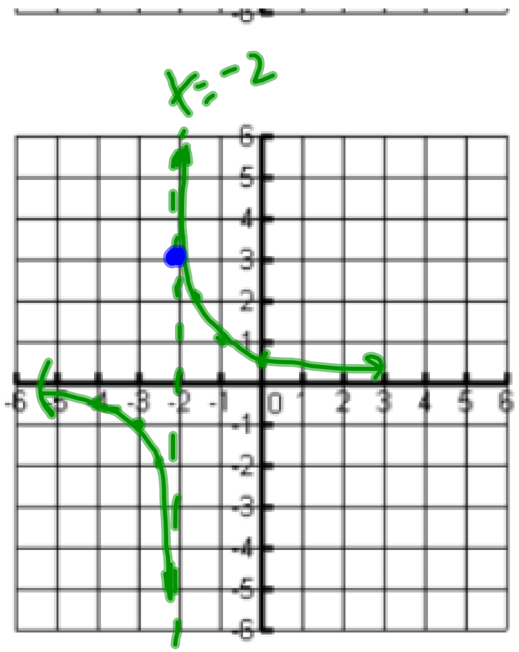
$\lim_{x \to -2} f(x)$ D.N.E.

Ex.2 $g(x) = \begin{cases} \frac{1}{x+2} & x \neq -2 \\ 3 & x = -2 \end{cases}$. Is g continuous at x = – 2?

Explain.

NO !

$\lim_{x \to -2} g(x)$ D.N.E.



Ex.3 $F(x) = \begin{cases} \dfrac{x^2-9}{x-3} & x \neq 3 \\ 5 & x = 3 \end{cases}$ Is F continuous at x = 3?

NO!

Explain.

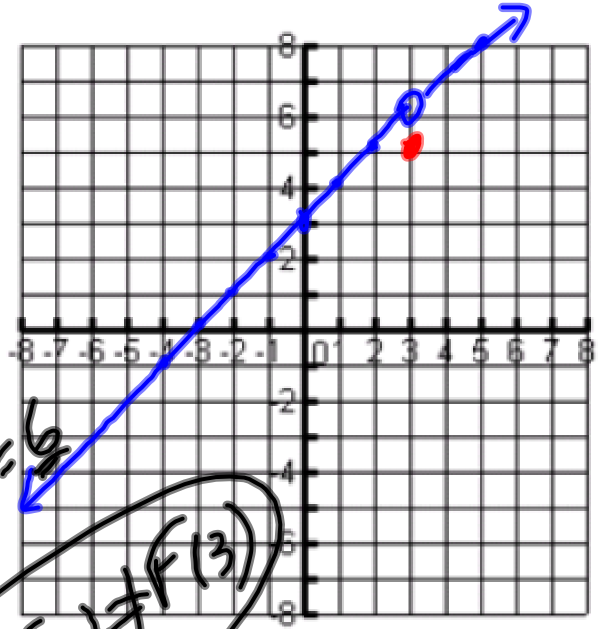
$f(x) = \dfrac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}$ $x \neq 3$

$F(x) = x+3 \,;\ x \neq 3$

i) $F(3) = 5$

ii) $\lim\limits_{x \to 3} F(x) = 6$

iii) $\lim\limits_{x \to 3} F(x) \neq F(3)$ *



To make F continuous at x = 3, how could the function be redefined?

$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & x \neq 3 \\ \underline{\underline{6}} & x = 3 \end{cases}$ ↔ (FILLS IN THE HOLE)

Ex,4 $G(x) = \dfrac{1}{\sqrt{2x-5}}$. For what values of x is G not continuous? Explain.

$x \le \frac{5}{2}$. G IS UND. THERE

DEN ≤ 0

$2x-5 \le 0$

$2x \le 5$

$x \le \frac{5}{2}$

O.T.C.

- CORRECT DAYS 1-7
- DO DAY 8
- MEMORIZE (& UNDERSTAND) THE FORMAL DEFINITIONS:
  - LIMIT
  - CONTINUITY
  - INTUITIVE DEFN. OF CONTINUITY

6 POINTS