

THE SLOPE GIFT

The slope of a line is a number, usually a fraction, that indicates “how slanted” a line is. Sometimes the slope of a line is a positive fraction (when slanting uphill), sometimes a negative fraction (when slanting downhill), sometimes zero (when horizontal), and sometimes the slope is undefined, that is, the line has no slope (when vertical).

IMPORTANT

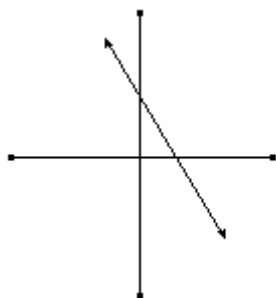
When deciding the slope (or slant) of a line, always read from left to right.

I. Fill in the blanks with the correct word or words:

1. The slope of any horizontal line, \leftrightarrow , will always be _____.
2. The slope of a line that slants uphill (from left to right, of course), \nearrow , will be a _____ fraction.
3. The slope of a line that slants downhill (from left to right, of course), \searrow , will be a _____ fraction.
4. The slope of any vertical line, \updownarrow , will always be _____ or _____.

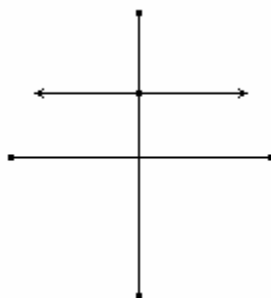
II. For each of the following lines, write a complete sentence telling whether the slope of the line is positive, negative, zero, or undefined (NO SLOPE), and tell why.

EX. 1

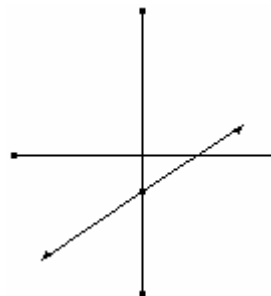


The slope of the line is _____
 negative because the line _____
 slants downhill. _____

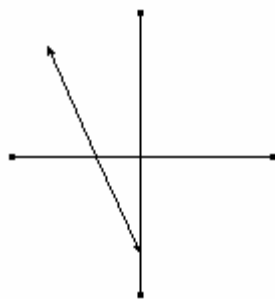
5.



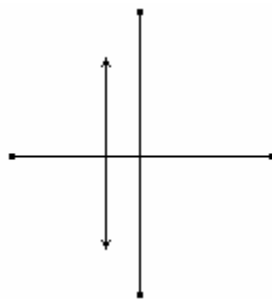
6.



7.



8.



III. So far we have only looked at the “sign” of the slope of a line. Now let us consider how you get the actual fraction. We will use the following symbols as a shorthand notation:

m stands for the word “slope”

Δ is the Greek letter, delta, that stands for the words “change in”

The concept of slope has several interpretations. Some of them are listed below:

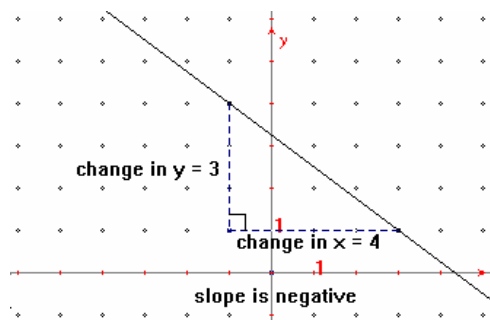
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = m$$

IV. To find the slope of a line graphically, you first decide if the slope is zero, undefined, positive, or negative – just based on appearance.

- If the slope is positive or negative, pick two points on the line and draw a right triangle (see example on the next page).
- The length of the vertical leg is the “change in y ” or Δy .
- The length of the horizontal leg is the “change in x ” or Δx .
- Then use the definition: $m = \frac{\Delta y}{\Delta x}$

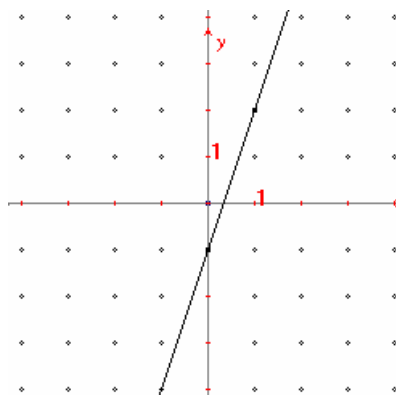
FIND THE SLOPE OF EACH OF THE FOLLOWING LINES GRAPHICALLY:

EX. 2

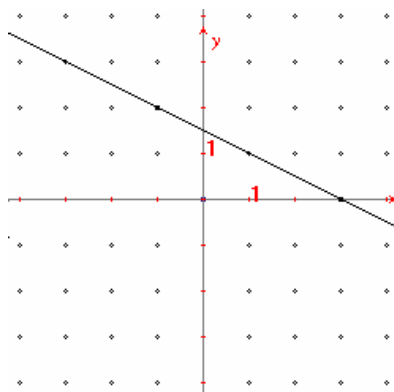


$$m = \frac{\Delta y}{\Delta x} = -\frac{3}{4}$$

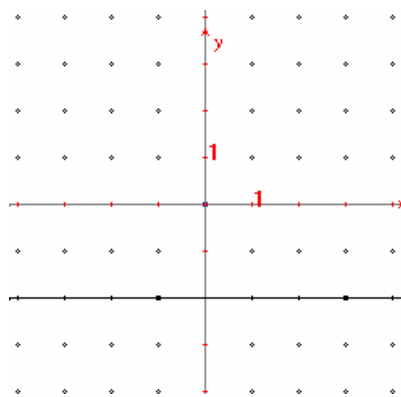
9.



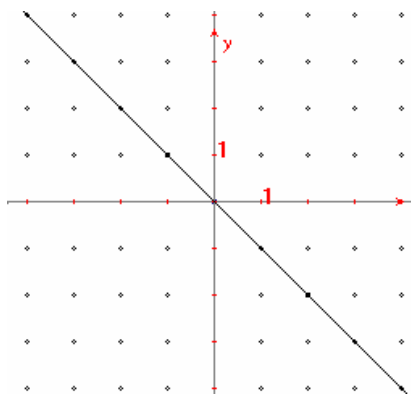
10.



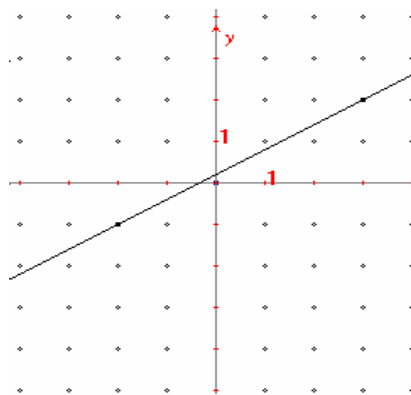
11.



12.



13.

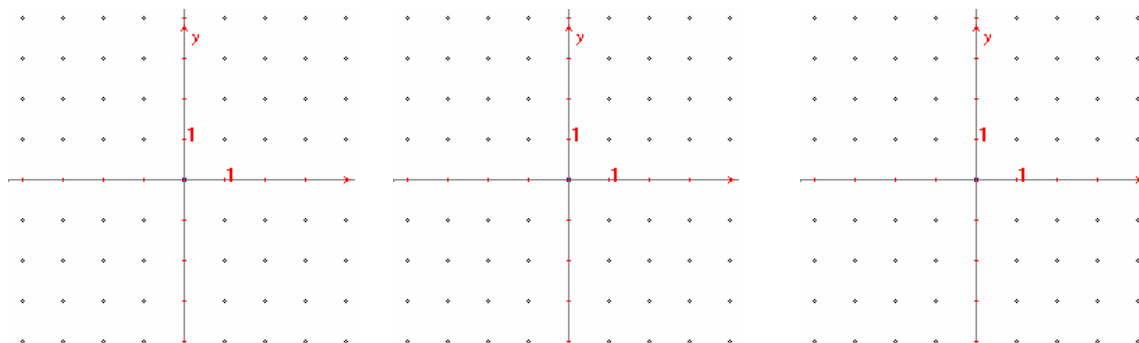


14 – 16. Find the slope of the line containing the two given points graphically:

14. $(-2, 3)$ $(1, -2)$

15. $(3, 2)$ $(-3, -1)$

16. $(2, -1)$ $(-3, -1)$



VI. In order to work with the slope formula that we will be using, you **MUST** be able to compute with signed numbers accurately – either using mental math or paper and pencil – no calculators!!!

17 – 28. Simplify the following expressions in lowest terms (no mixed numbers)

17. $\frac{-4}{-6}$

18. $\frac{16}{-6}$

19. $\frac{0}{5}$

20. $\frac{-16}{-12}$

21. $\frac{4}{0}$

22. $\frac{12-4}{5-7}$

23. $\frac{1-(-1)}{2-6}$

24. $\frac{-2-3}{4-4}$

25. $\frac{-3-3}{3-4}$

26. $\frac{-7-2}{-1-5}$

27. $\frac{-5-(-5)}{-4-3}$

28. $\frac{4-(-6)}{-3-1}$

NOTE: You must understand how to do these types of calculations in order to be successful here.

VII. FINDING THE SLOPE OF A LINE BY FORMULA

Since “change in” means to subtract, the following formula should make sense.

- The slope of a line containing two points: (x_1, y_1) and (x_2, y_2) is found by:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

On the next page, read example 3. Once you understand it, finish example 4.

EX. 3 Find the slope of the line containing: (5, -2) (-3, 4)

$$\text{Let } x_1 = 5, x_2 = -3$$

$$y_1 = -2, y_2 = 4$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - (-2)}{-3 - 5}$$

$$= \frac{6}{-8}$$

$$m = \underline{\underline{-\frac{3}{4}}}$$

\therefore The line slants downhill

EX. 4 Find the slope of the line containing: (-1, 3) (-2, -4)

29 – 36. On your paper, find the slope of the line containing the given points by formula only:

29. (5, 2) (9, 3) 30. (-3, 0) (4, -2) 31. (-6, -5) (4, 7) 32. (3, -2) (4, -2)

33. (1, -2) (-2, 3) 34. (-3, -1) (3, 2) 35. (6, 2) (6, -1) 36. (2, -1) (-3, -1)

37. To see that you obtain the same answer both graphically and by formula, compare your answer to:

a) number 33 with number 14 on page 4.

b) number 34 with number 15 on page 4.

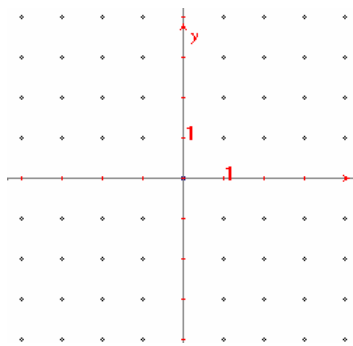
c) number 36 with number 16 on page 4.

VII. DRAWING LINES WITH GIVEN SLOPE

For each of the following, draw a line through the given point that has the given slope.

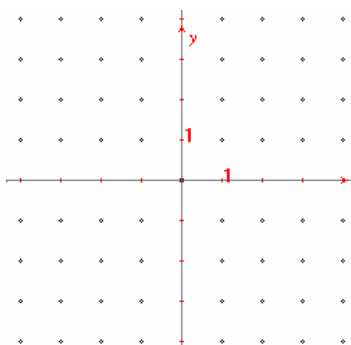
EX. 5

$$(-1, 2) \quad m = -\frac{3}{4}$$



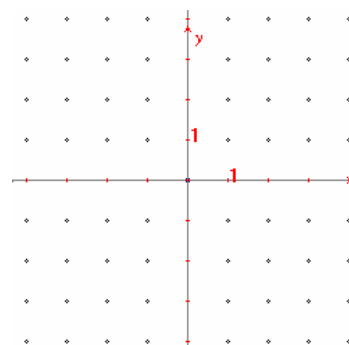
EX. 6

$$(-2, -3) \quad m = 2$$



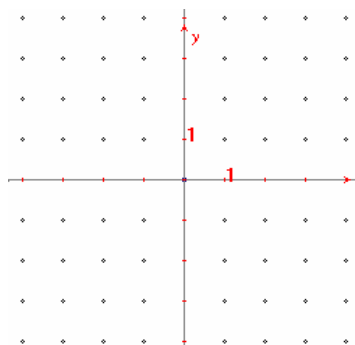
EX. 7

$$(-3, 1) \quad m = -3$$

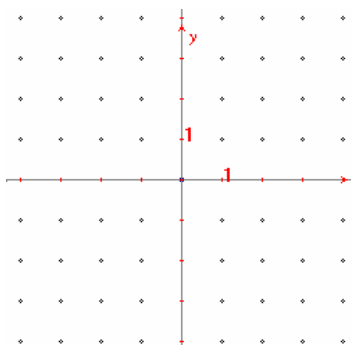


38 – 43. Draw a line through the given point that has the given slope.

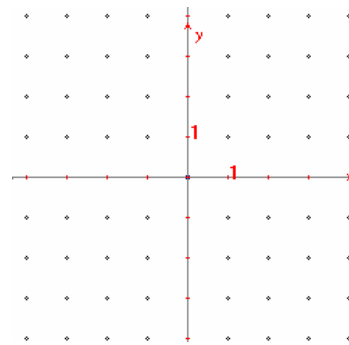
38. $(1, 4) \quad m = -\frac{2}{3}$



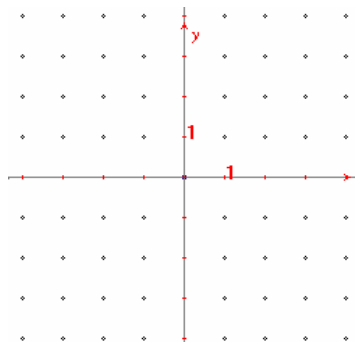
39. $(-1, 1) \quad m = 2$



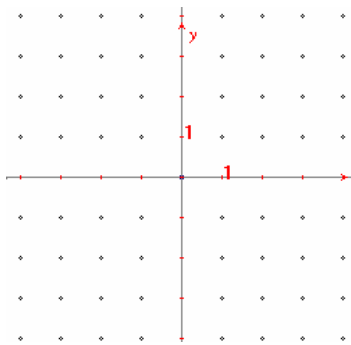
40. $(-1, 3) \quad \text{no slope}$



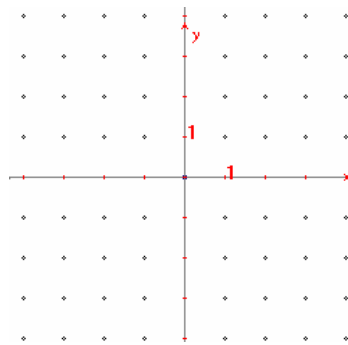
41. $(-2, -3) \quad m = -\frac{1}{2}$



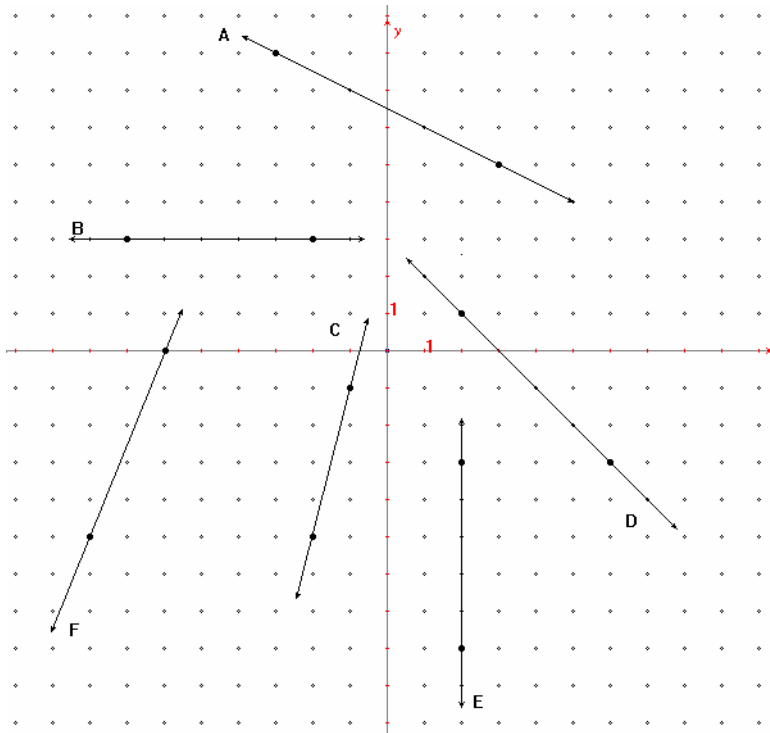
42. $(-1, -2) \quad m = \frac{1}{4}$



43. $(3, 1) \quad m = 0$



IX. EXERCISES



44. Find the slope of each line:

A)

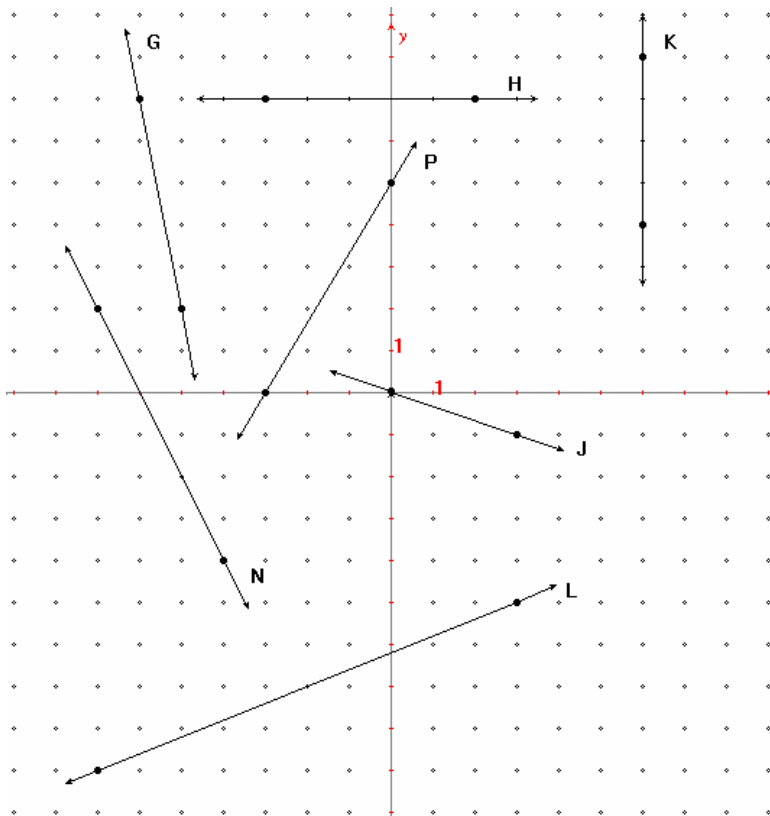
B)

C)

D)

E)

F)



45. Find the slope of each line

G)

H)

J)

K)

L)

N)

P)

46. Through the given point, draw **and label** a line with the given slope:

a) $(-9, 2)$ $m = 4$

b) $(0, -2)$ $m = -\frac{1}{2}$

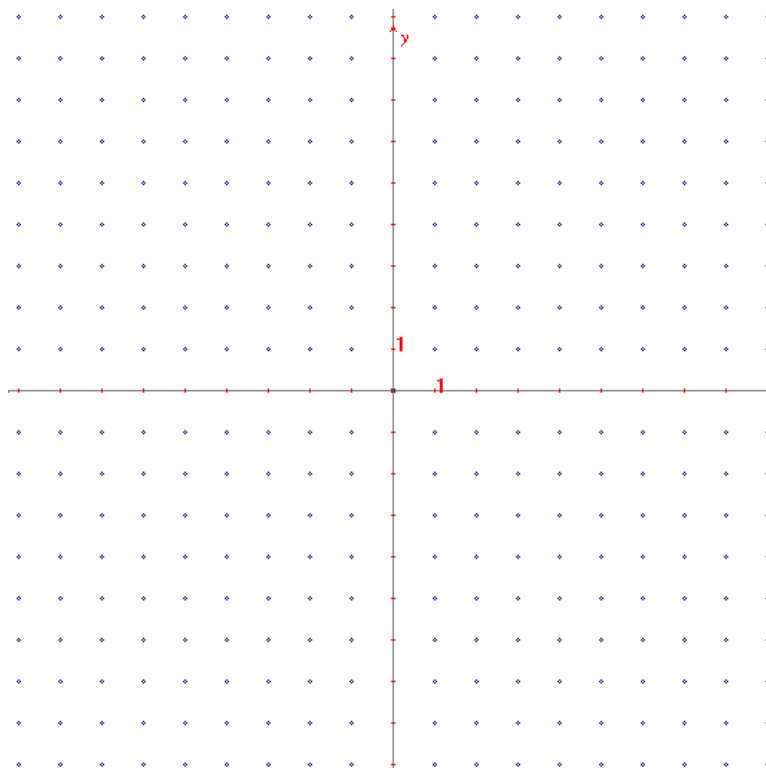
c) $(-7, -3)$ no slope

d) $(3, 5)$ $m = -1$

e) $(-5, 0)$ $m = \frac{3}{5}$

f) $(2, 7)$ $m = 0$

g) $(3, -6)$ $m = -\frac{2}{5}$



47. Through the given point, draw **and label** a line with the given slope:

h) $(1, 7)$ $m = -\frac{2}{3}$

i) $(1, -7)$ $m = 0$

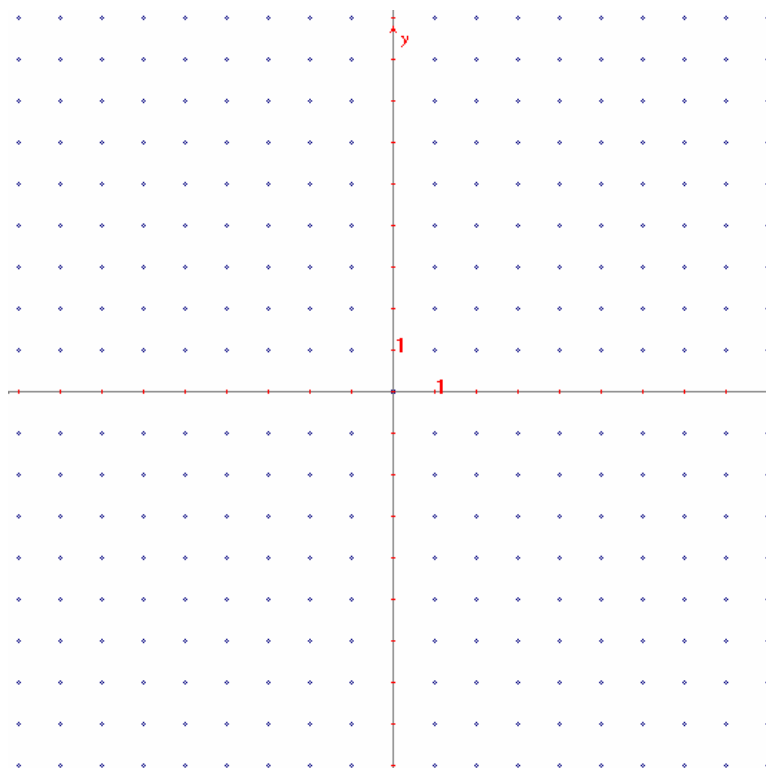
j) $(-6, 0)$ $m = 7$

k) $(-7, -7)$ $m = \frac{2}{3}$

l) $(-2, 5)$ $m = -1$

n) $(2, -3)$ $m = -\frac{1}{5}$

q) $(7, -1)$ no slope



KEY

THE SLOPE GIFT

The slope of a line is a number, usually a fraction, that indicates "how slanted" a line is. Sometimes the slope of a line is a positive fraction (when slanting uphill), sometimes a negative fraction (when slanting downhill), sometimes zero (when horizontal), and sometimes the slope is undefined, that is, the line has no slope (when vertical).

IMPORTANT

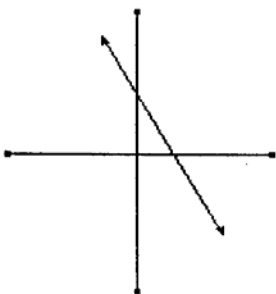
When deciding the slope (or slant) of a line, always read from left to right.

I. Fill in the blanks with the correct word or words:

- The slope of any horizontal line, \leftrightarrow , will always be ZERO.
- The slope of a line that slants uphill (from left to right, of course), \nearrow , will be a POSITIVE fraction.
- The slope of a line that slants downhill (from left to right, of course), \searrow , will be a NEGATIVE fraction.
- The slope of any vertical line, \updownarrow , will always be UNDEFINED or NO SLOPE.

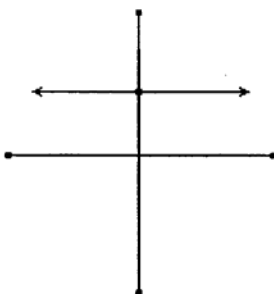
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EX. 1



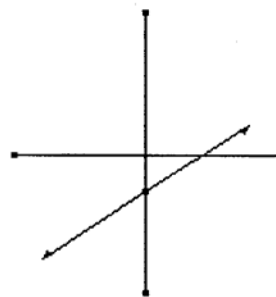
The slope of the line is
negative because the line
slants downhill.

5.



THE SLOPE OF THE
LINE IS ZERO
BECAUSE IT IS
HORIZONTAL

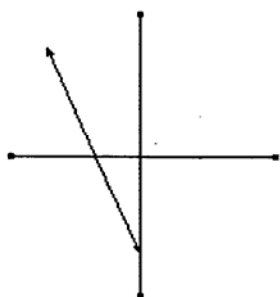
6.



THE SLOPE OF THE
LINE IS POSITIVE BECAUSE
IT SLANTS UPHILL

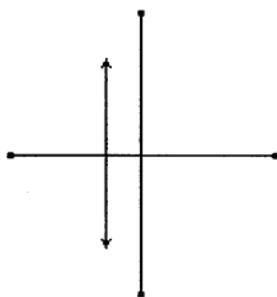
KEY

7.



THE SLOPE OF THE
LINE IS NEGATIVE
BECAUSE IT SLANTS
DOWN HILL.

8.



THE LINE HAS
NO SLOPE BECAUSE
IT IS VERTICAL.

III. So far we have only looked at the “sign” of the slope of a line. Now let us consider how you get the actual fraction. We will use the following symbols as a shorthand notation:

m stands for the word “slope”

Δ is the Greek letter, delta, that stands for the words “change in”

The concept of slope has several interpretations. Some of them are listed below:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = m$$

IV. To find the slope of a line graphically, you first decide if the slope is zero, undefined, positive, or negative – just based on appearance.

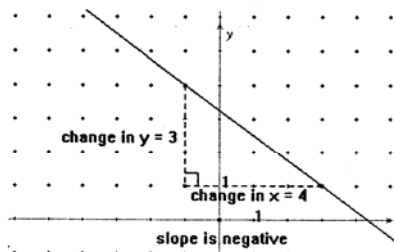
- If the slope is positive or negative, pick two points on the line and draw a right triangle (see example on the next page).
- The length of the vertical leg is the “change in y ” or Δy .
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- Then use the definition: $m = \frac{\Delta y}{\Delta x}$

KEY

3

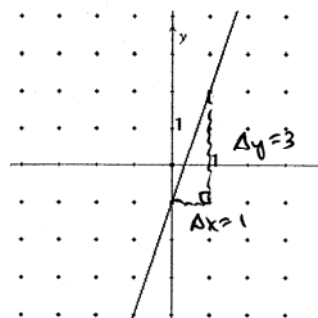
FIND THE SLOPE OF EACH OF THE FOLLOWING LINES GRAPHICALLY:

EX. 2



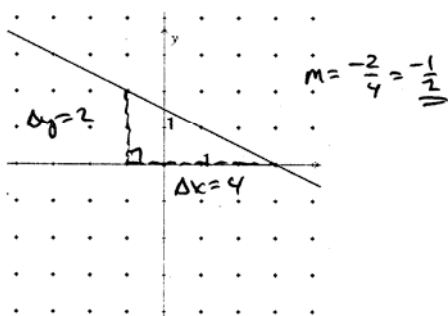
$$m = \frac{\Delta y}{\Delta x} = -\frac{3}{4}$$

9.



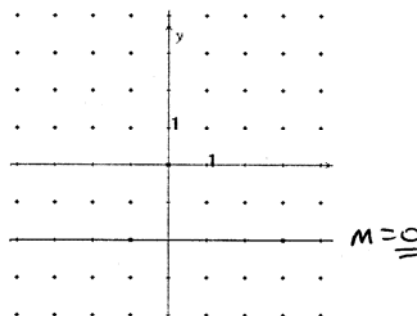
$$m = +\frac{3}{1} = 3$$

10.



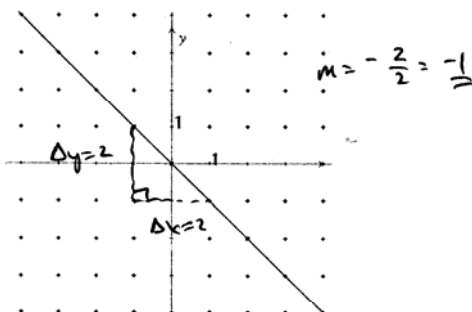
$$m = -\frac{2}{4} = -\frac{1}{2}$$

11.



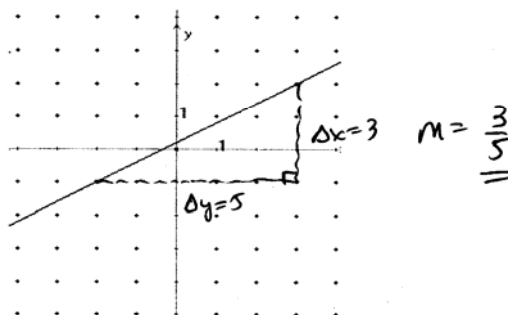
$$m = 0$$

12.



$$m = -\frac{2}{2} = -1$$

13.



$$m = \frac{5}{3}$$

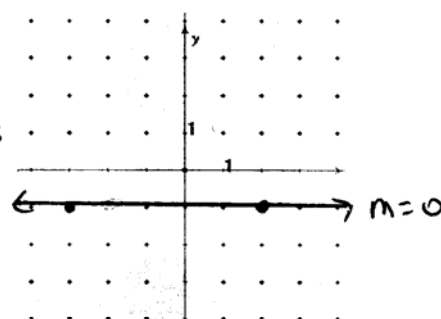
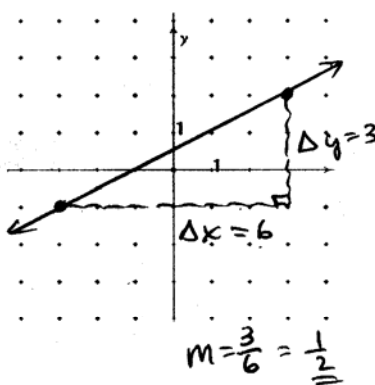
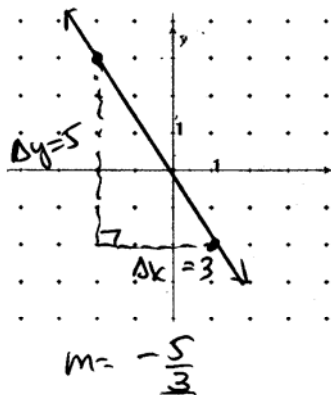
KEY

14 – 16. Find the slope of the line containing the two given points graphically:

14. $(-2, 3)$ $(1, -2)$

15. $(3, 2)$ $(-3, -1)$

16. $(2, -1)$ $(-3, -1)$



VI. In order to work with the slope formula that we will be using, you **MUST** be able to compute with signed numbers accurately – either using mental math or paper and pencil – no calculators!!!

17 – 28. Simplify the following expressions in lowest terms (no mixed numbers)

$$17. \frac{-4}{-6} = \frac{2}{3} \quad 18. \frac{16}{-6} = -\frac{8}{3} \quad 19. \frac{0}{5} = 0 \quad 20. \frac{-16}{-12} = \frac{4}{3} \quad 21. \frac{4}{0} = \text{UNDEFINED}$$

$$22. \frac{12-4}{5-7} = -\frac{4}{2} \quad 23. \frac{1-(-1)}{2-6} = -\frac{1}{2} \quad 24. \frac{-2-3}{4-4} = \text{UNDEFINED} \quad 25. \frac{-3-3}{3-4} = \frac{6}{1}$$

$$26. \frac{-7-2}{-1-5} = \frac{3}{2} \quad 27. \frac{-5-(-5)}{-4-3} = 0 \quad 28. \frac{4-(-6)}{-3-1} = -\frac{5}{2}$$

NOTE: You must understand how to do these types of calculations in order to be successful here.

VII. FINDING THE SLOPE OF A LINE BY FORMULA

Since “change in” means to subtract, the following formula should make sense.

- The slope of a line containing two points: (x_1, y_1) and (x_2, y_2) is found by:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

On the next page, read example 3. Once you understand it, finish example 4.

KEY

5

EX. 3 Find the slope of the line containing: (5, -2) (-3, 4)

$$\text{Let } x_1 = 5, x_2 = -3$$

$$y_1 = -2, y_2 = 4$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - (-2)}{-3 - 5}$$

$$= \frac{6}{-8}$$

$$m = \underline{\underline{-\frac{3}{4}}}$$

∴ The line slants downhill

EX. 4 Find the slope of the line containing: (-1, 3) (-2, -4)

29 – 36. On your paper, find the slope of the line containing the given points by formula only:

29. (5, 2) (9, 3) $\frac{1}{4}$ 30. (-3, 0) (4, -2) $-\frac{2}{7}$ 31. (-6, -5) (4, 7) $\frac{6}{5}$ 32. (3, -2) (4, -2) 0
33. (1, -2) (-2, 3) $-\frac{5}{3}$ 34. (-3, -1) (3, 2) $\frac{1}{2}$ 35. (6, 2) (6, -1) No Slope 36. (2, -1) (-3, -1) 0

37. To see that you obtain the same answer both graphically and by formula, compare your answer to:

- a) number 33 with number 14 on page 4. ✓
- b) number 34 with number 15 on page 4. ✓
- c) number 36 with number 16 on page 4. ✓

Key

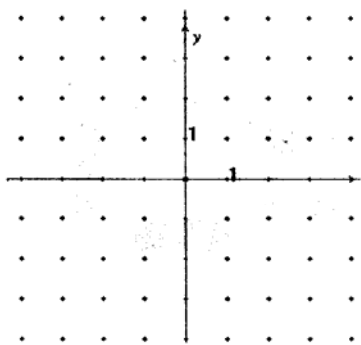
6

VII. DRAWING LINES WITH GIVEN SLOPE

For each of the following, draw a line through the given point that has the given slope.

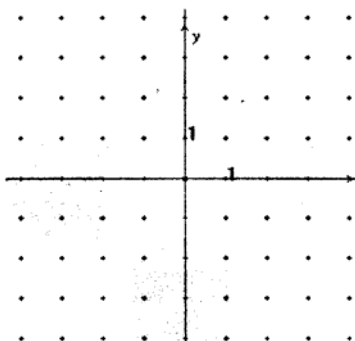
EX. 5

$$(-1, 2) \quad m = -\frac{3}{4}$$



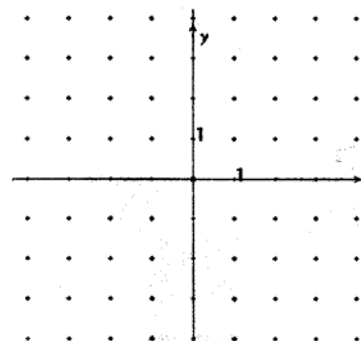
EX. 6

$$(-2, -3) \quad m = 2$$



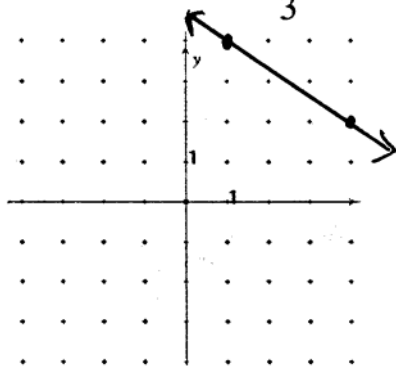
EX. 7

$$(-3, 1) \quad m = -3$$

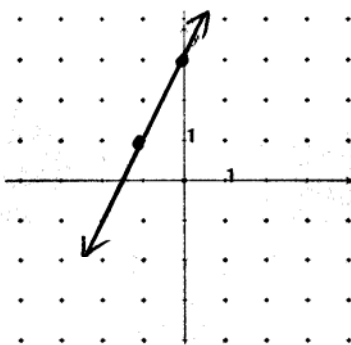


38 – 43. Draw a line through the given point that has the given slope.

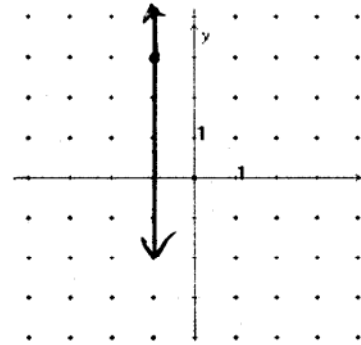
$$38. (1, 4) \quad m = -\frac{2}{3}$$



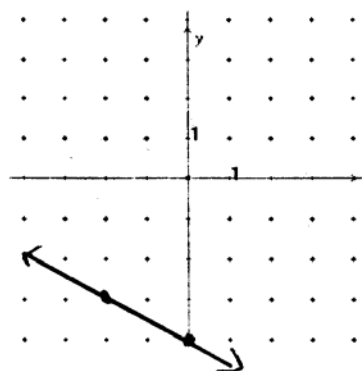
$$39. (-1, 1) \quad m = 2$$



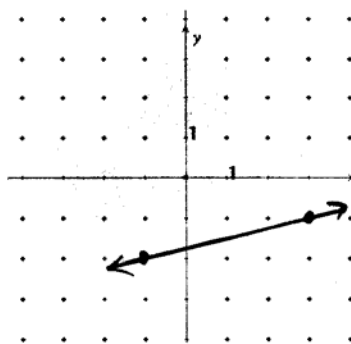
$$40. (-1, 3) \quad \text{no slope}$$



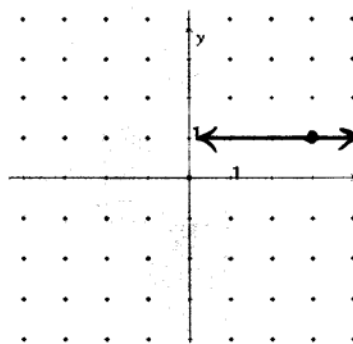
$$41. (-2, -3) \quad m = -\frac{1}{2}$$



$$42. (-1, -2) \quad m = \frac{1}{4}$$

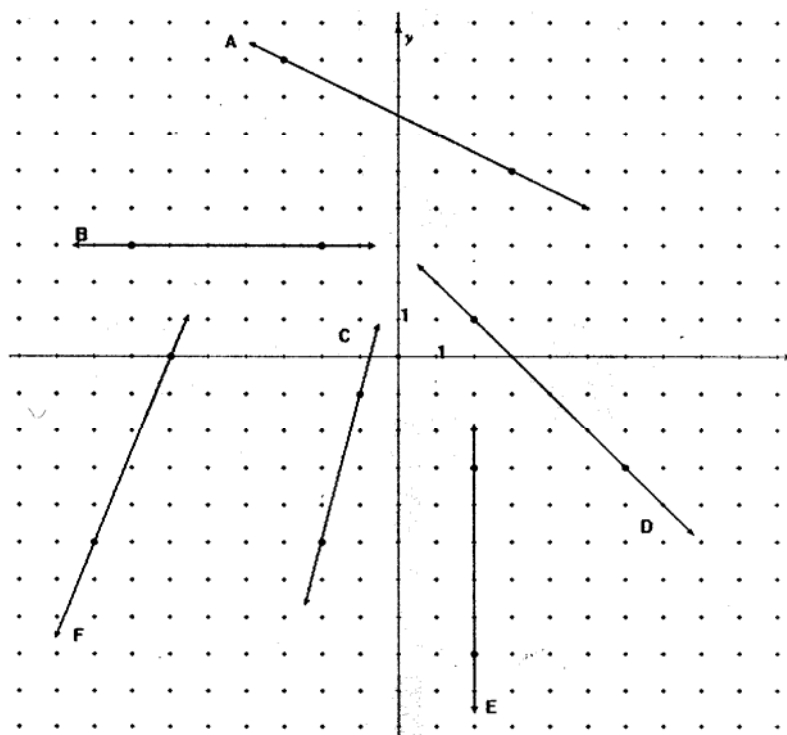


$$43. (3, 1) \quad m = 0$$



KEY 7

IX. EXERCISES



44. Find the slope of each line:

A) $-\frac{1}{2}$

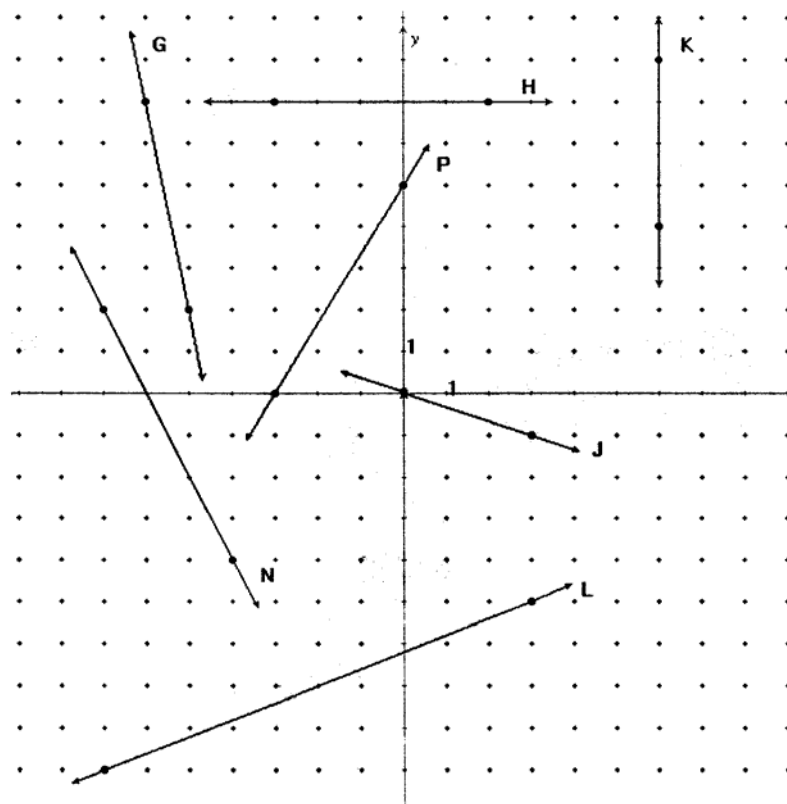
B) 0

C) 4

D) -1

E) NO SLOPE (UNDEFINED)

F) $\frac{5}{2}$



45. Find the slope of each line

G) -5

H) 0

J) $-\frac{1}{3}$

K) NO SLOPE (UNDEFINED)

L) $\frac{2}{5}$

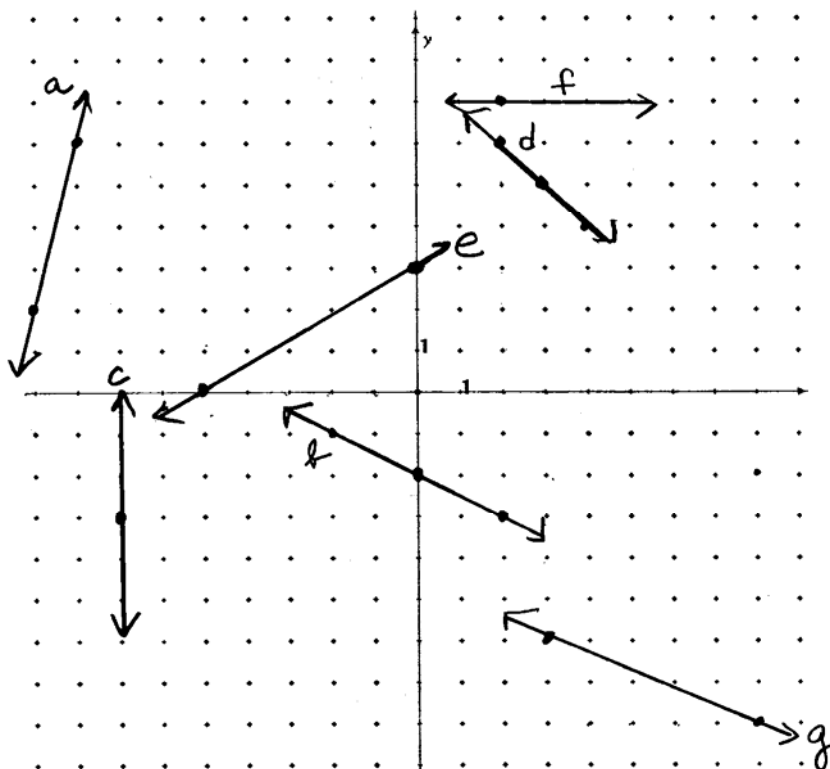
N) -2

P) $\frac{5}{2}$

KEY 8

46. Through the given point, draw **and label** a line with the given slope:

- a) $(-9, 2)$ $m = 4$
 b) $(0, -2)$ $m = -\frac{1}{2}$
 c) $(-7, -3)$ no slope
 d) $(3, 5)$ $m = -1$
 e) $(-5, 0)$ $m = \frac{3}{5}$
 f) $(2, 7)$ $m = 0$
 g) $(3, -6)$ $m = -\frac{2}{5}$



47. Through the given point, draw **and label** a line with the given slope:

- h) $(1, 7)$ $m = -\frac{2}{3}$
 i) $(1, -7)$ $m = 0$
 j) $(-6, 0)$ $m = 7$
 k) $(-7, -7)$ $m = \frac{2}{3}$
 l) $(-2, 5)$ $m = -1$
 n) $(2, -3)$ $m = -\frac{1}{5}$
 q) $(7, -1)$ no slope

