

THE PHONE BILL



DETAILED CHARGES

ACIS

For Questions Regarding AT&T Charges Call 1-800-222-0300.

330 75

NOV 4, 1996

CURRENT CHARGES

AT&T Invoice Charges For Period Ending OCT 27, 1996

Long Distance

No.	Date	Time	Place Called	Number	Code	Min	Amount
1	9-28	728P	COLUMBUS	OH 614 688-	N	1	.14
2	10-01	635P	COLUMBUS	OH 614 688-	E	16	2.64
3	10-06	306P	COLUMBUS	OH 614 688-	E	25	4.13
4	10-13	913P	COLUMBUS	OH 614 688-	E	1	.18
5	10-24	952P	COLUMBUS	OH 614 688-	E	4	.66
Total Itemized Calls							

Calling Card Calls

No.	Date	Time	Place Called	Number	Code	Min	Amount
1	10-15	914P	YOUNGSTOWN	OH 330 757	EC	15	4.45
		FROM	COLUMBUS	OH 614 621			
6	9-28	805P	YOUNGSTOWN	OH 330 757-	NC	16	4.22
		FROM	COLUMBUS	OH 614 621-			
7	9-30	604P	YOUNGSTOWN	OH 330 757-	EC	5	2.05
		FROM	COLUMBUS	OH 614 621-			
8	10-01	522P	YOUNGSTOWN	OH 330 757-	EC	1	1.09
		FROM	COLUMBUS	OH 614 621-			
9	10-08	854P	YOUNGSTOWN	OH 330 757	EC	32	8.53
		FROM	COLUMBUS	OH 614 621-			
10	10-13	535P	YOUNGSTOWN	OH 330 757-	EC	1	1.09
		FROM	COLUMBUS	OH 614 621			
11	10-13	957P	YOUNGSTOWN	OH 330 757-	EC	26	7.09
		FROM	COLUMBUS	OH 614 621-			

This is a copy of part of my long distance phone bill for November, 1996. My son had just started his first quarter at The Ohio State University and through a miscommunication (pardon the pun), he was using the calling card instead of using the long distance service (UNITS) that the university supplied. While I was paying the phone bill, I wondered if there was an easily recognizable relationship between the cost of the call and the length of the call in minutes.

1. Using the calling card data, you will notice that 7 calls are listed. Should we use all 7 calls for data? Discuss.

2. Which data should be used?

Copy and complete the table below using the appropriate data, but write the data in ascending order by the number of minutes.

# min.	cost
--------	------

3. Using your graphing calculator, clear two consecutive lists (L_1 , L_2) or create two lists (like MINITS, COST). Enter the data into the lists: # of minutes into L_1 and cost into L_2 .

4. Turn off or clear all $y =$ equations. Turn off all Stat Plots except the one you intend to use. Create an appropriate window and record its dimensions. Then using Stat Plot, plot the data and investigate.

5. What type of mathematical relationship do you notice?

6. Let's see if we can find a linear equation in two variables that can serve as a model for this data. What will we need to compute first and how will we do that?

7. Using your graphing calculator, find the slope and store its value into M .

8. Again, using your graphing calculator, find the y-intercept and store that value into B .

9. Enter this equation into y_1 .

10. Graph. Discuss what you observe.

11. Let's test the data to see how good our model equation is.

a) Evaluate y_1 at $x = 5$. What do you get?

b) Evaluate y_1 at $x = 32$. What do you get?

c) Discuss why these two values should match EXACTLY with the original data.

12. Now let's try the other data points.

- Evaluate y_1 at $x = 1$; Evaluate y_1 at $x = 15$; Evaluate y_1 at $x = 26$
- How close are these to the actual data?
- How good is our model?

13. A second way to check the data.

- On the graph screen, press TRACE. Notice that you are tracing along the PLOT 1 (P1: L_1, L_2) data. Press the right cursor arrow a few times to "trace through" the data.
- Once you get the point (32,8.53) you will notice that you can't continue to the right anymore, because there is no more data. To "jump" to the graph of y_1 , just press the down cursor arrow once.
- Since you are in trace mode, you can type in any value for x that is in the window's domain for x and it will go to that point on the graph of y_1 and display the corresponding y value. Do that for $x = 1, 5, 15, 26, 32$ and compare the "model equation" values for the cost with the actual data.

14. A third way to check the data is to create a table.

- Using TBLSET, have TblStart = 0, $\Delta Tbl = 1$.
To view the table: 2nd GRAPH (TABLE).
- Use the down cursor arrow to go through the values in the table and check with the original data.

15. You may want to view the graph and table at the same time.

One way is to be in Horizontal mode. To do this, press the MODE key and cursor down to the last row and highlight "Horiz"; then 2nd MODE (QUIT). Then press GRAPH; TRACE; Press down arrow to get y_1 ; Type in 30; 2nd GRAPH (TABLE); Cursor down through table until $x = 30$.

16. Another way to view the graph and table simultaneously is using the G - T mode (Graph - Table mode).

Press MODE key and cursor down to highlight G - T; 2nd MODE (QUIT); Press GRAPH; TRACE; Down arrow; 25

17. a) Discuss what the equation $y = .24x + .85$ means.

- What does the .24 stand for? Explain.
- What does the .85 stand for? Explain.
- Rewrite $y = .24x + .85$ using function notation, letting C represent the cost of a phone call as a function of time, T , in minutes.

18. a) Will the calculator generated regression equation obtain the same equation we did? Investigate and see.

- 2nd MODE(QUIT) to return to the home screen; STAT; CALC;
4 LinReg (ax+b); <Enter>; L_1, L_2, y_2 ; <Enter>

18c) Discuss what this means and how this compares to "your" regression equation.

d) Look at $y_2 =$. What is the equation listed for $y_2 = ?$ _____

e) Turn off y_1 ; GRAPH; TRACE; Down arrow once to get to y_2 .

19. Using the equation you obtained, answer the following questions:

a) How much should a call cost if the call lasts:

i. 10 minutes? ii. 20 minutes? iii. an hour? iv. a day?

NOTE: If you are trying to approximate data that lies *in between* values given, it is called **interpolation**. If you are trying to approximate data that is *outside or beyond* the values given, it is called **extrapolation**.

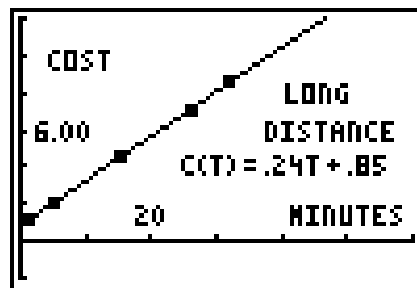
For b, c, d: Solve algebraically, but confirm graphically:

b) If a call costs \$6.13, how long was the call?

c) If a call costs \$13.09, how long was the call?

d) If a call costs \$20, how long was the call?

20. It is sometimes a good idea to label your graph with a title and appropriate units so that it can be understood more easily. By using the DRAW features, entitle the graph "Long Distance". Label the x-axis with "Minutes", the y-axis with "Cost". Show where 20 minutes would be, show where \$6.00 would be. Also label the equation " $C(T) = .24T + .85$ ". The final result should look something like the following:



Once you are satisfied with your picture, save it by:

Leave the graph screen by typing 2nd MODE(QUIT); 2nd PRGM(DRAW); STO; 1 StorePic; <Enter>; 1 (or any number from 0 to 9); <Enter>

21. You may also want to save your data into a program as follows:

PRGM; NEW; <Enter>; Name=PHONDATA <Enter>;

: ClrList L1, L2

: {1, 5, 15, 25, 32} -> L1 To get this type: 2nd STO(RCL) L1 <Enter>

: {1.09, 2.05, 4.45, 7.09, 8.53} -> L2 To get this type: 2nd STO(RCL) L2

<Enter>

:

2nd MODE(QUIT)

THE PHONE BILL TEACHER'S ANSWERS AND EXPLANATIONS

1. No. Only use the data with the same code -- EC. NC would be at a different rate probably.

2.

# of min.	cost
1	1.09
5	2.05
15	4.45
26	7.09
32	8.53

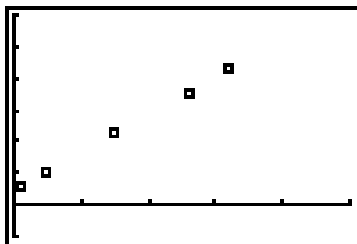
3.

L1	L2	L3
1	1.09	-----
5	2.05	
15	4.45	
26	7.09	
32	8.53	
-----	-----	
L2(6) =		

This is what your STAT Edit screen should look like.

4. X [0, 50]₁₀ Y [-2, 12]₂

```
WINDOW
Xmin=0
Xmax=50
Xscl=10
Ymin=-2
Ymax=12
Yscl=2
Xres=1
```



These are the dimensions for the window that I suggest. I like to leave room under the x-axis for tracing.

5. It appears linear. Suggestion: On view screen, use spaghetti (or linguini because it's flat) to "connect" data points to emphasize the linear relationship.

6. By picking 2 appropriate points, find the slope. I suggest using points 2 and 5. Caution students that it is not a good idea to use two consecutive points and discuss why.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

7. You can do this at least 2 ways depending upon the level of your students.

1 Way "Basic":

Enter data "by hand" and compute the slope.

(8.53-2.05)/(32-5) → M
 .24

2nd Way "Advanced"

(Recommended)

Encourage your students to use symbolic representation to solve:

(L₂(5)-L₂(2))/(L₁(5)-L₁(2)) → M
 .24

This way is recommended because it really emphasizes that slope is the "change in y over the change in x" and it uses subscripted variables.

8. Again two ways are possible. Depending on the level of your students:

1 Way "Basic": Using either of the two points, for example, point 2: (5, 2.05) and the value for the slope just found, $m = .24$, substitute into $y = mx + b$.

$$\begin{aligned} 2.05 &= (.24)(5) + b \\ 2.05 &= 1.20 + b \\ \underline{.85} &= b \end{aligned}$$

2nd Way "Advanced" (Recommended): Starting with $y = mx + b$, solve that equation for b : $y - mx = b$. Use the second point to substitute, but use subscripted variables as follows:

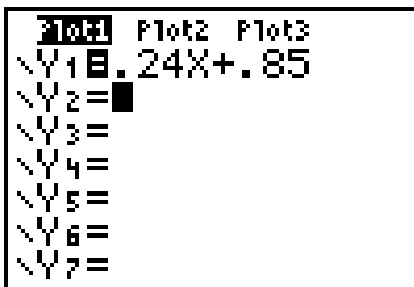
L₂(2) - M*L₁(2) → B
 .85

***VARIATION:** Have students also find the y-intercept this way (using subscripted variables), but using point 5 to verify that it doesn't matter which of the two points you use -- you get the SAME VALUE for b:

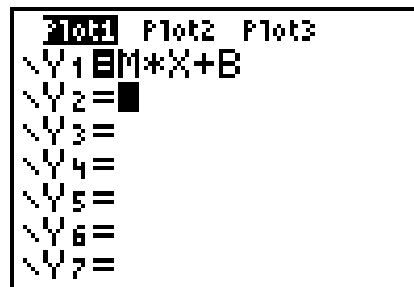
$$L_2(5) - M * L_1(5) + B$$

.85

9. This can be done numerically:

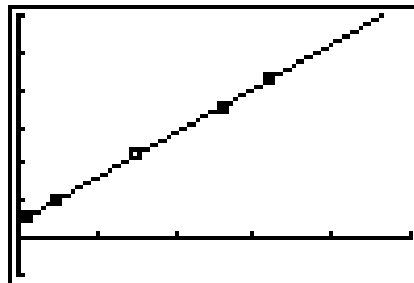


or symbolically:



This is more appropriate -- especially if m or b were to be non-terminating decimals. This would eliminate rounding and be more accurate. It also eliminates the error of entering the values for m and b incorrectly.

10. This looks pretty good. Discuss why the line doesn't "fill in" the 3rd box (*because of the pixels*):



11.

$Y_1(5)$	2.05
$Y_1(32)$	8.53

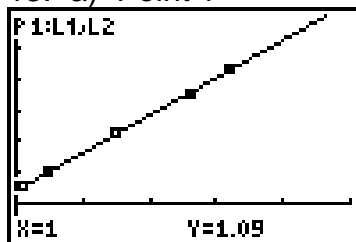
Notice that the TI-83 will let you use function notation to evaluate the functions stored in the $y =$ menu.

- a. 2.05 b. 8.53
 c. These were the 2 points we used! They *have* to work. Any two points chosen would create a linear equation that those 2 chosen points would satisfy.

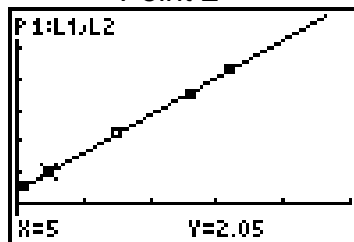
12. a) $y_1(1) = 1.09$; $y_1(15) = 4.45$; $y_1(20) = 7.09$

b) They match EXACTLY. c) Our model is exactly what AT&T uses.

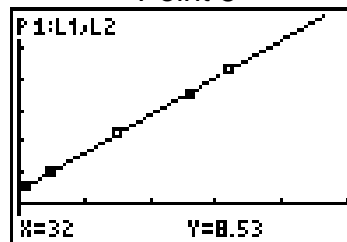
13. a) Point 1



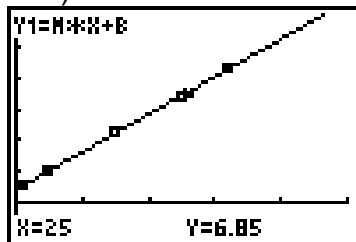
Point 2



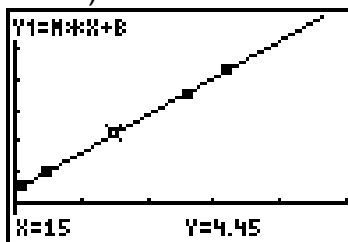
Point 5



13b)



13c)



Shown is the trace for $x = 15$, $y = 4.45$. This is the point that doesn't look "filled in." They still match exactly.

14.a)

TABLE SETUP	
TblStart=0	
Δ Tbl=1	
Indent:	Auto Ask
Depend:	Auto Ask

14.b)

X	Y_1	
-1	.61	
0	.85	
1	1.09	
2	1.33	
3	1.57	
4	1.81	
5	2.05	

$Y_1 = M * X + B$

X	Y_1	
26	7.09	
27	7.33	
28	7.57	
29	7.81	
30	8.05	
31	8.29	
32	8.53	

X=32

15.

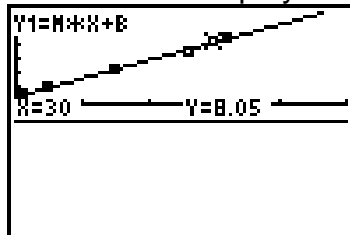
MODE Horiz

```

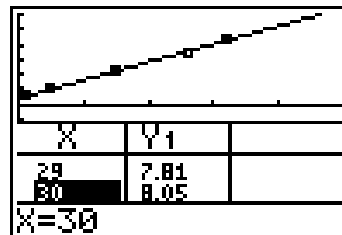
Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full G-T

```

Horiz MODE display



Horiz MODE with table



16.

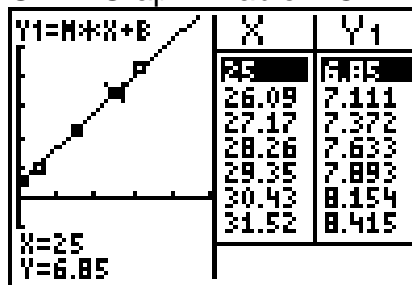
MODE G - T

```

Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T

```

G - T Graph - Table MODE



With the class, discuss that the table is not using $\Delta Tbl = 1$ unless you specify it. It is trying to correspond to values on the graph.

17. a, b, c) .24 is the slope, which represents the cost per minute once the call has been connected. .85 is the y-intercept which represents the initial cost of connecting the call.

d) $C(T) = .24T + .85$ NOTE: The discussion of this is one of the most important parts of this exercise.

18. a) YES.

b) STAT CALC

```

EDIT TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7:QuartReg

```

Store result in Y_2

```

LinReg(ax+b) L1,
L2, Y2

```

The regression equation

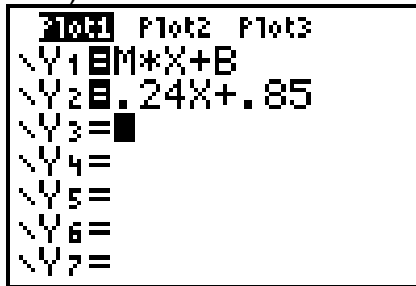
```

LinReg
y=ax+b
a=.24
b=.85
r^2=1
r=1

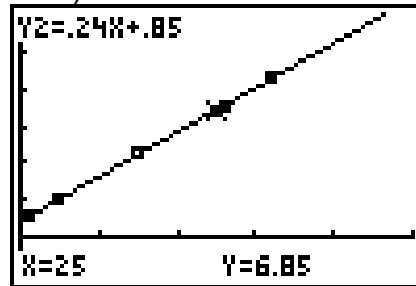
```

18 c) Exactly the same. $r^2 = 1$ & $r = 1$ indicates that the correlation is exact.

18 d)



18 e)

19. a) $y_2(10) = 3.25$; $y_2(20) = 5.65$; $y_2(60) = 15.25$; $y_2(24*60) = 346.45$

19. b) Solve algebraically:

$$.24x + .85 = 6.13$$

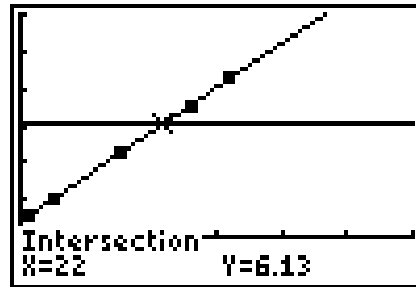
$$.24x = 5.28$$

$$x = 22 \text{ min.}$$

Confirm graphically:

Graph $y_3 = 6.13$

Use 2nd TRACE (CALC) 5 Intersect

19. c) $x = 51 \text{ min.}$ 19. d) $x \approx 79.8 \approx 80 \text{ min.}$

20. Explain that the "picture" is stored in Pic 1 (or another single digit from 0 to 9 that you choose) and the picture can be recalled by:

- 1) Turn off all $y =$ equations
- 2) 2nd ZOOM (FORMAT); Axes Off; 2nd MODE (QUIT)
- 3) 2nd PRGM (DRAW); STO; 2: RecallPic; 1; <ENTER>

21. Explain that in order to put that phone bill data (PHONDATA) back into list L_1 , L_2 , do the following:

PRGM; EXEC; PHONDATA; <ENTER>; <ENTER>

Check L_1 , L_2 by STAT; EDIT; L_1 , L_2 will have been cleared of all previous data and restored with the 5 data points.