TRIG ESSENTIALS

ANSWER KEY 1 OF 4

The following is a brief summary of the fundamentals of trigonometry. This is a minimum of what *ANYONE* who has studied trig should have a good grasp of. Not only are you expected to know and understand the following, you must be able to ***USE AND APPLY*** it in order to be successful in the remainder of the course. Also, for those of you who may be going on in math, science, or computers, this is the trig that you must know to be successful in calculus. I urge you to:

1) Correctly and neatly complete this gift.
2) Study and understand it.
3) Keep it for future reference (*you'll thank yourself for this someday*).

I. Complete the **GRAPHS** on the attached page first. Use the graphs to help answer the questions as to period, undefined values, quadrants positive or negative, domain, range,...

II. EXACT SPECIAL VALUES

A. Families

$\sin\frac{\pi}{6} = \frac{1}{2}$ $\quad \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\quad \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad \cos\frac{\pi}{3} = \frac{1}{2}$

$\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$ $\quad \tan\frac{\pi}{4} = 1$ $\quad \tan\frac{\pi}{3} = \sqrt{3}$

B. Axis Values $\quad 0 \quad \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2}$

$\sin 0 = 0$
$\cos 0 = 1$
$\tan 0 = 0$

$\sin\frac{\pi}{2} = 1$
$\cos\frac{\pi}{2} = 0$
$\tan\frac{\pi}{2} = \text{UND.}$

$\sin\pi = 0$
$\cos\pi = -1$
$\tan\pi = 0$

$\sin\frac{3\pi}{2} = -1$
$\cos\frac{3\pi}{2} = 0$
$\tan\frac{3\pi}{2} = \text{UND.}$

III. In what QUADRANTS are the functions positive (POS) or negative (NEG)?

	POS	NEG		POS	NEG
sin	I, II	III, IV	csc	I, II	III, IV
cos	I, IV	II, III	sec	I, IV	II, III
tan	I, III	II, IV	cot	I, III	II, IV

IV. Fundamental Period

	Fund. Period	Equation to illustrate	
sin	2π	$\sin(x+2k\pi)=\sin x$	$x\in\mathbb{R}$, $k\in J$
cos	2π	$\cos(x+2k\pi)=\cos x$	
tan	π	$\tan(x+k\pi)=\tan x$	

V. Definitions

	Definition	Undefined Values
tan x =	$\frac{\sin x}{\cos x}$	$x\neq(2k+1)\frac{\pi}{2}$
cot x =	$\frac{\cos x}{\sin x}$	$x\neq k\pi$
sec x =	$\frac{1}{\cos x}$	$x\neq(2k+1)\frac{\pi}{2}$
csc x =	$\frac{1}{\sin x}$	$x\neq k\pi$

VI. Odd or Even Function?

	Complete	Odd or Even?
sin(-x) =	$-\sin x$	ODD
cos(-x) =	$\cos x$	EVEN
tan(-x) =	$-\tan x$	ODD

The graph of an even function is symmetrical with respect to THE y-AXIS.

The graph of an odd function is symmetrical with respect to THE ORIGIN.

VII. Identities

$\sin(a\pm b)=$ $\sin a\cos b\pm\cos a\sin b$.

$\cos(x\pm y)=$ $\cos x\cos y\mp\sin x\sin y$

$\sin 2x=$ $2\sin x\cos x$

$\sin\frac{1}{2}x=$ $\pm\sqrt{\frac{1}{2}(1-\cos x)}$

$\cos 2x=$ $2\cos^2 x-1$

$\cos 2x=$ $1-2\sin^2 x$

$\cos 2x=$ $\cos^2 x-\sin^2 x$

$\cos\frac{1}{2}x=$ $\pm\sqrt{\frac{1}{2}(1+\cos x)}$

List all 5 equivalent forms of the Pythagorean identity:

$\sin^2 x+\cos^2 x=1$

$\sin^2 x=1-\cos^2 x$

$\cos^2 x=1-\sin^2 x$

$\tan^2 x+1=\sec^2 x$

$\cot^2 x+1=\csc^2 x$

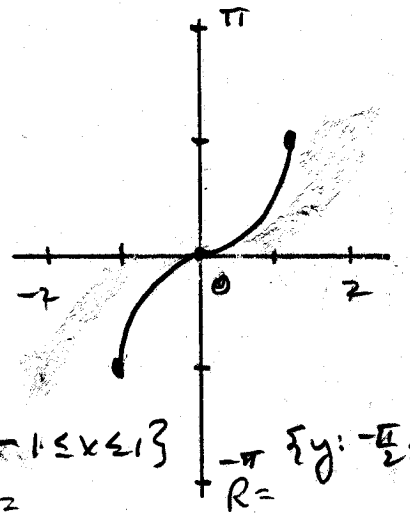
VIII. Inverse Trig functions

Complete: An inverse of a function is formed by "JUST" SWITCHING THE x- AND y- COORDINATES.

What is true about the graph of a relation and the graph of its inverse?
THEY ARE SYMMETRICAL ABOUT THE LINE $y = x$.

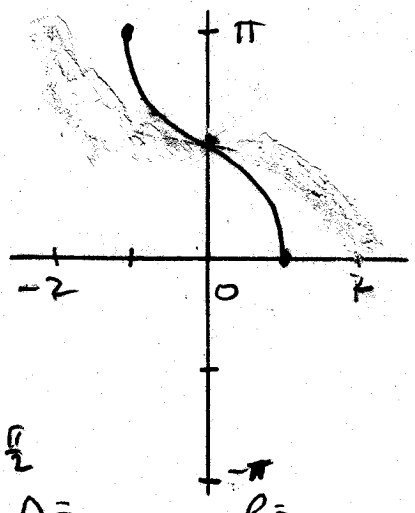
Label the units appropriately on the axes.

Graph and state the domain and range for $y = Sin^{-1}x$



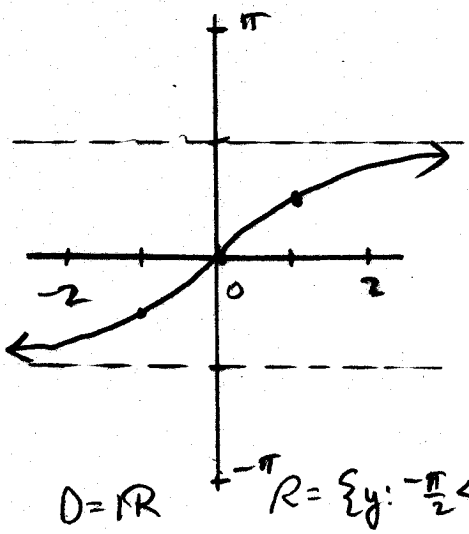
$\{x: -1 \le x \le 1\}$ $\{y: -\frac{\pi}{2} \le y \le \frac{\pi}{2}\}$
D= R=

Graph and state the domain and range for $y = Cos^{-1}x$



D= $\{x: -1 \le x \le 1\}$ R= $\{y: 0 \le y \le \pi\}$

Graph and state the domain and range for $y = Tan^{-1}x$



D= $\mathbb{R}$ R= $\{y: -\frac{\pi}{2} < y < \frac{\pi}{2}\}$

Compute exactly.

1. a) $Sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ b) $Sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ c) $Arcsin\ 1 = \frac{\pi}{2}$ d) $Arcsin\ (-1) = -\frac{\pi}{2}$

2. a) $Cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ b) $Cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ c) $Arccos\ 1 = 0$ d) $Arccos\ (-1) = \pi$

3. a) $Tan^{-1}\ 1 = \frac{\pi}{4}$ b) $Tan^{-1}\ (-1) = -\frac{\pi}{4}$ c) $Arctan\ \sqrt{3} = \frac{\pi}{3}$ d) $Arctan\ (-\sqrt{3}) = -\frac{\pi}{3}$

4 – 6. Solve: a) $0 \le x < 2\pi$ (specific solutions) b) general solutions

4. $\sin 2x = \sin x$

a) $\{0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}\}$

b) $\{x: x = k\pi\}$ $\cup \{x: x = \frac{\pi}{3} + 2k\pi\}$ $\cup \{x: x = \frac{5\pi}{3} + 2k\pi\}$

5. $2\cos^2 x - \cos x = 1$

a) $\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

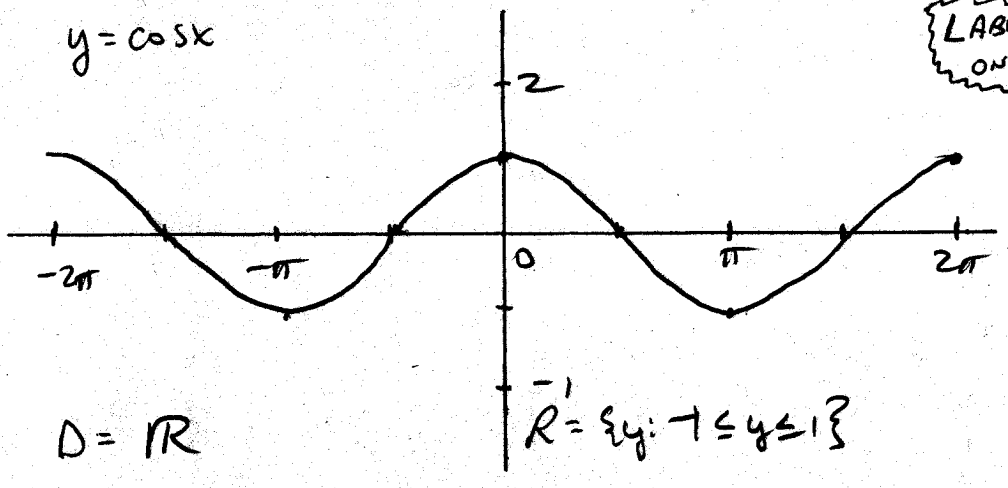
b) $\{x: x = 2k\pi\}$ $\cup \{x: x = \frac{2\pi}{3} + 2k\pi\}$ $\cup \{x: x = \frac{4\pi}{3} + 2k\pi\}$

6. $\tan^2 x - \tan x = 0$

a) $\{0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}\}$

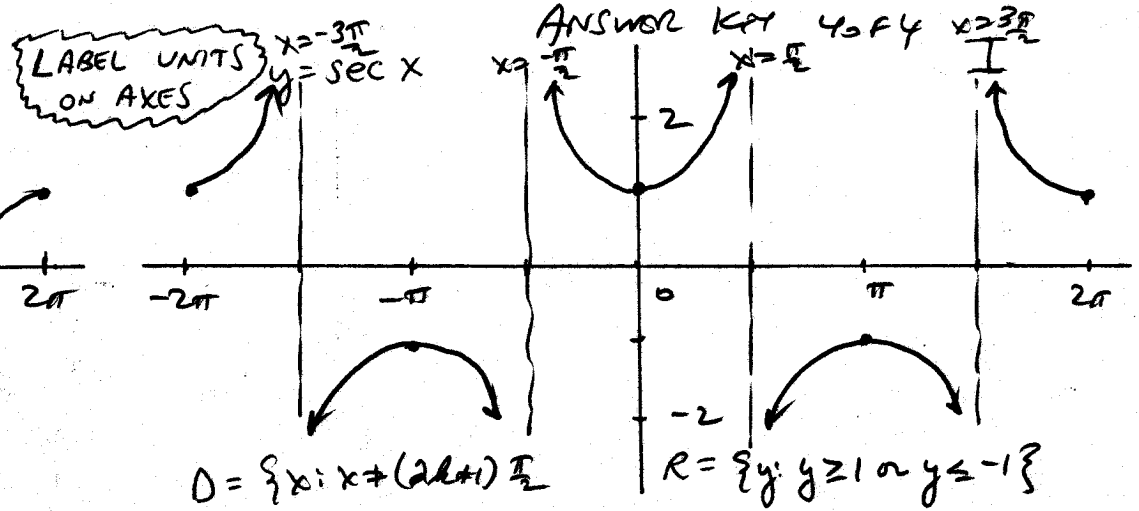
b) $\{x: x = k\pi\}$ $\cup \{x = \frac{\pi}{4} + k\pi\}$

ANSWER KEY 4 of 4



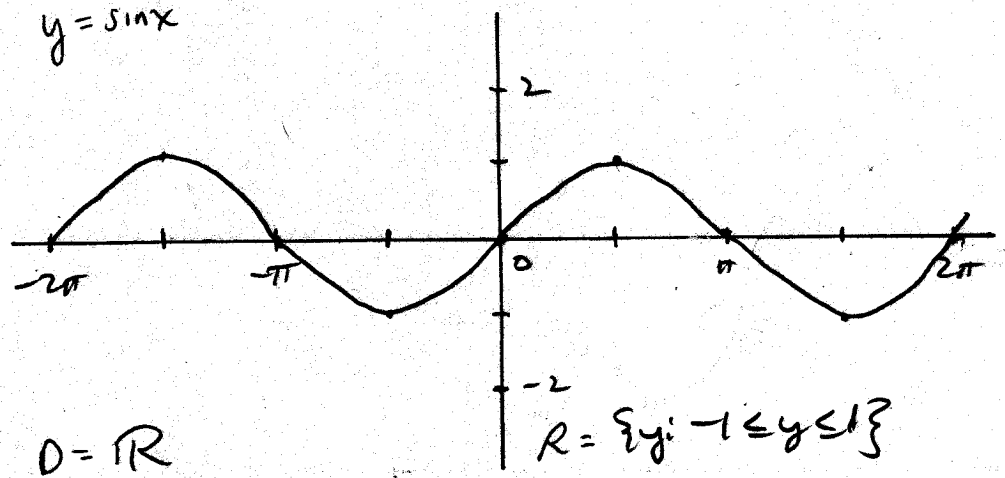
$D = \mathbb{R}$

$R = \{y : -1 \le y \le 1\}$



$D = \{x : x \neq (2k+1)\frac{\pi}{2}\}$

$R = \{y : y \ge 1 \text{ or } y \le -1\}$



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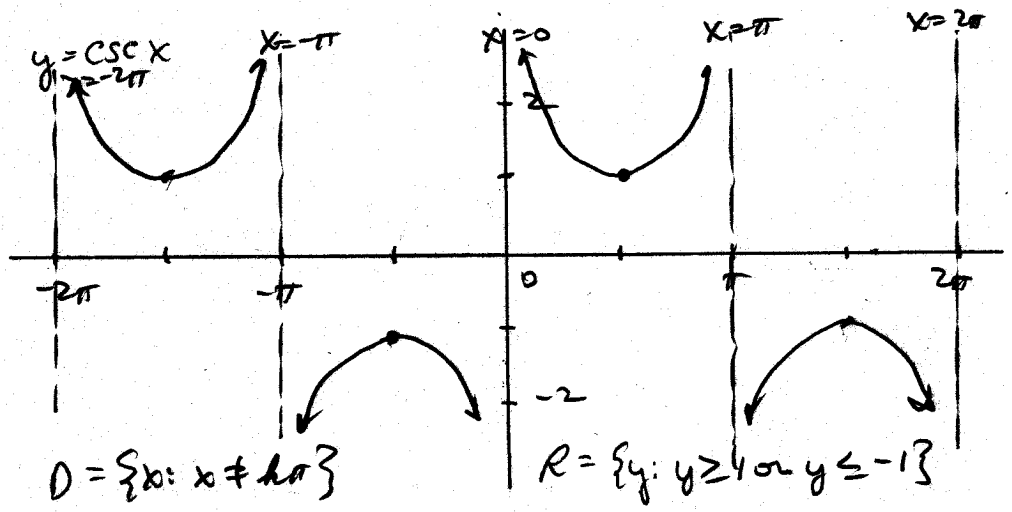

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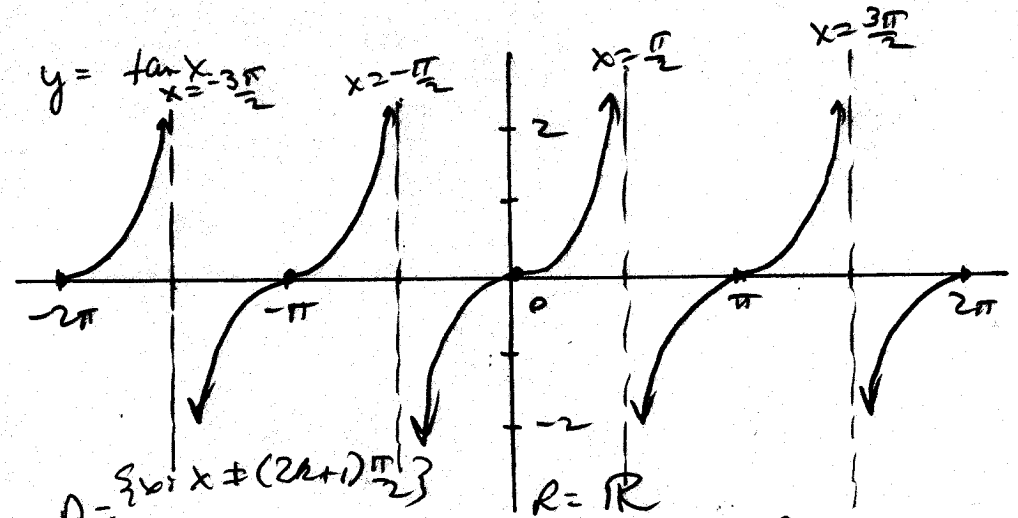

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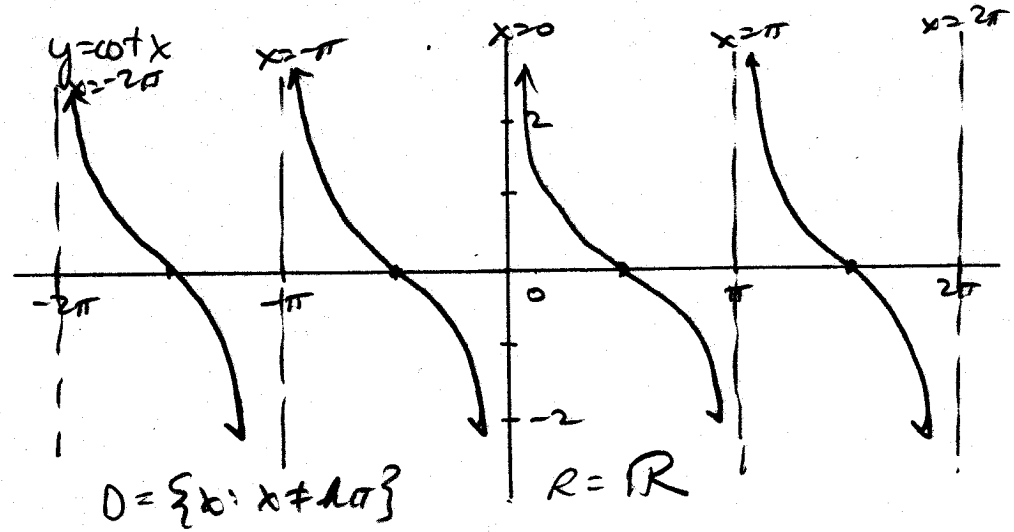

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