TI-Nspire CAS Ideas as to How to Incorporate into the High School Classroom

Integrate CAS with Graphing, Geometry, Spreadsheets, and Notes – So Powerful!

NOTE: Selected screen shots of 3 activities

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Slopes of Tangent Lines to Parabolas

Student Version

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In this activity, you will investigate the slopes of lines tangent to several parabolas. You will be asked to do some of the activity on paper (supplied) and the rest of the activity on Nspire. Please look for patterns as we proceed.

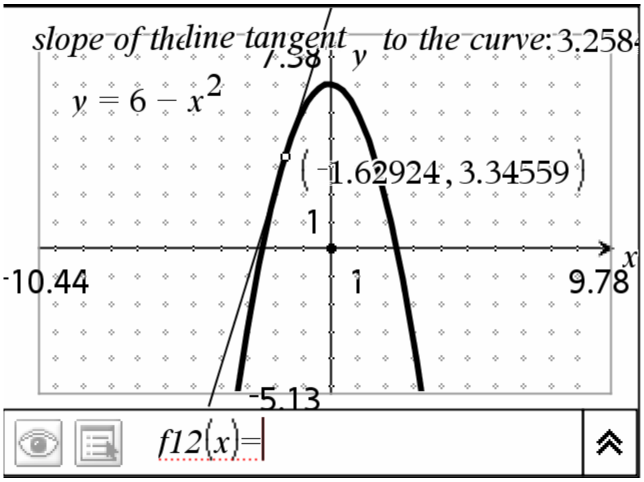
To illustrate what we are to accomplish, you will be asked to go to the next page and do the following:

1. With the pointer, grab the point on the curve.
2. As you move the point along the parabola, try to approximate the slope of the tangent line to the parabola.

It is best to approximate this when the x–coordinate of the point is an integer –– positive, negative, or even zero. (Also try to get the y–coordinate of the point to be an integer also.) Round the

It is best to approximate this when the x–coordinate of the point is an integer –– positive, negative, or even zero. (Also try to get the y–coordinate of the point to be an integer also.) Round the slope to the nearest whole number.

Then compare your approximation with the slope supplied at the top of the page. Use the grid to assist you as necessary.

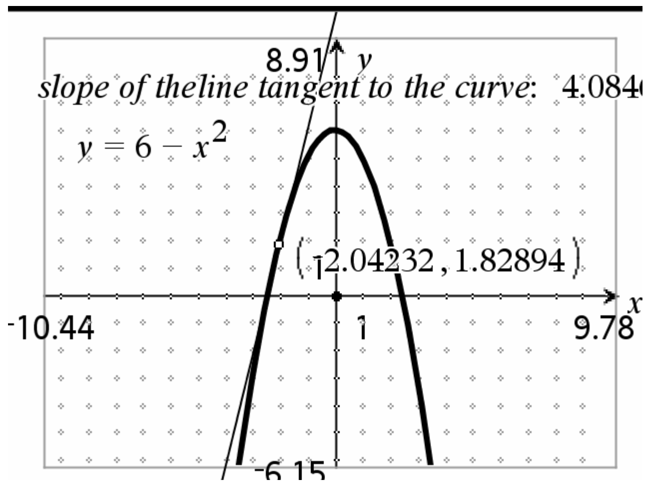


1a) Now that you have an understanding of what we want to accomplish, do Activity Page 1 part a (on the paper supplied).

When you have completed part a, then continue to the next page of this Nspire file.

1b) Using the graph on the next page, with the pointer, grab the point and record the slope of the tangent line to the parabola in the third column of the table on Activity Page 1 at each of the five values of x.

Compare your (student) answers with the calculator's answers. They may not match perfectly, but they should be reasonably close.



NOTE: the following directions are also listed on the activity page.

On the next page, you will send data to two lists.

The x–coordinate will be sent to the first list and the corresponding slope of the tangent line to the curve will be sent to the second list.

Please do this for 8 to 10 different values.

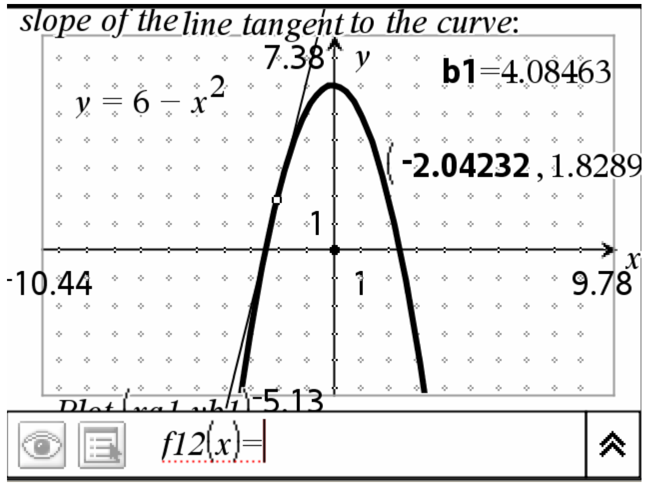
You will use the manual data capture. This is accomplished by:

1. Click in the graph screen.
2. Place the pointer so that the point on the tangent line is selected. Move the point so that the x–coordinate of the point is approximately −2.5.
3. Type the ctrl key and the decimal point key simultaneously.
4. This should place the x–coordinate

point key simultaneously.

4. This should place the x−coordinate into the first list and the corresponding slope into the second list. Grab the point and move it so that the x−coordinate is approximately −2.0. Redo step 3 above.

5. Continue to move the point and redo step 3 until between 8 and 10 data values are collected.



	A (...)	B (...)	C	D	E	F	G
◆	=capt	=capt					
1	-2.2...	4.4...					
2	-2.0...	4.1...					
3	-1.8...	3.7...					
4	-1.6...	3.2...					
5	-1.2...	2.4...					
6	-.69...	1.3...					

A1 | =-2.23489240071

NOTE: the following directions are also listed on the activity page.

Now that the data is collected, we want to plot it on the same pair of axes as the original function. To do this:

1. In the graph part of the screen on the previous page, show the entry line.

2. From the Graph Type menu, select the Scatter Plot option. Click on the arrow in the x box and select xa1.

3. Click on the arrow in the y box and select yb1.

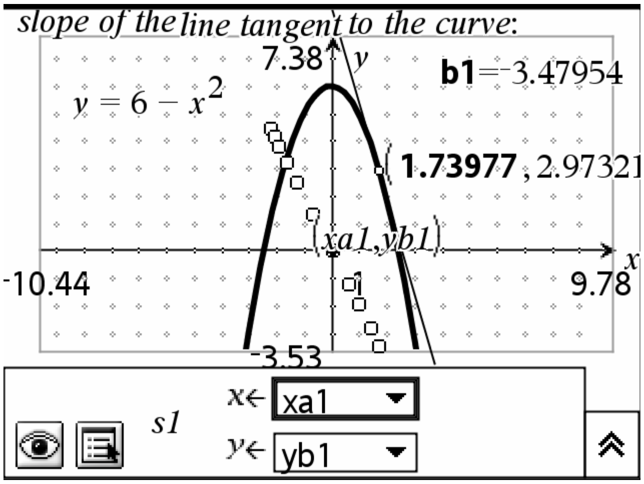
4. The plotted points should appear.

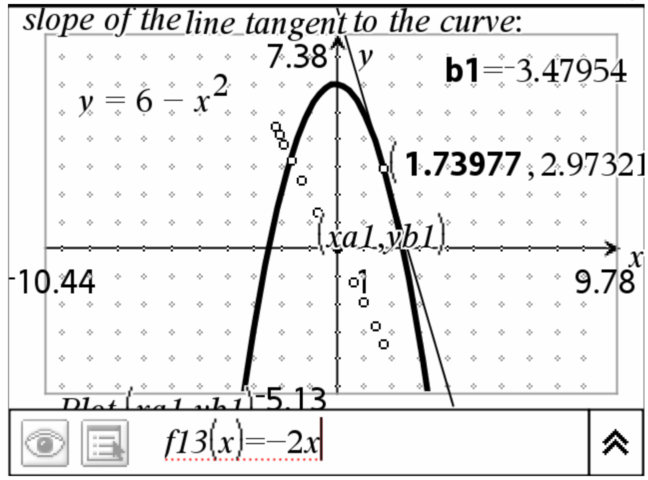
5. Now try to decide the equation of best fit that contains these 8 to 10 points.

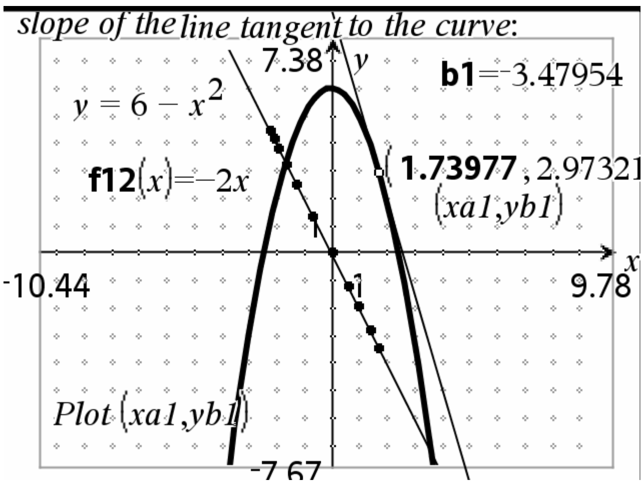
6. From the Graph Type menu, select the Function option. Type your equation of best fit into the next available f(x) = and then graph it to see just how good

best fit that contains these 8 to 10 points.

6. From the Graph Type menu, select the Function option. Type your equation of best fit into the next available f(x) = and then graph it to see just how good
of a fit it actually is for the plotted points. Compare this equation with the one you obtained on Activity Page 1, part d.







We will continue the investigations. On the next several screens will be graphs of various parabolas, each with a tangent line. You will be asked to manually collect data into a spreadsheet: the x-coordinate of the point on the curve and its corresponding slope of the tangent line. Then plot that data on the same pair of axes as the original curve.

The following was created by Steve Arnold of Australia:

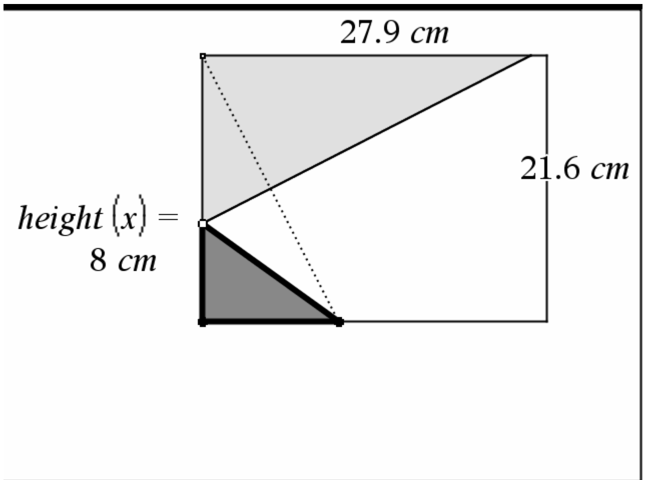
AREAS AND PAPER FOLDING

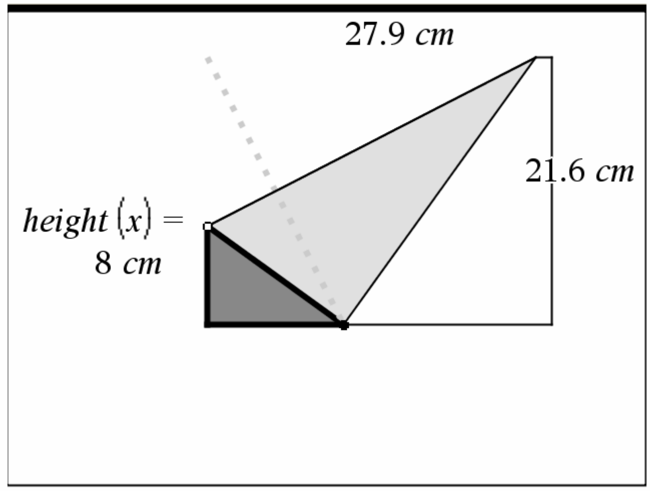
Measurement, data, graphs, algebra and geometry...

Question

Fold the top left corner of a sheet of paper to touch the opposite (long) side to form a triangle in the bottom left corner.
What fold gives the largest triangle?

Answer





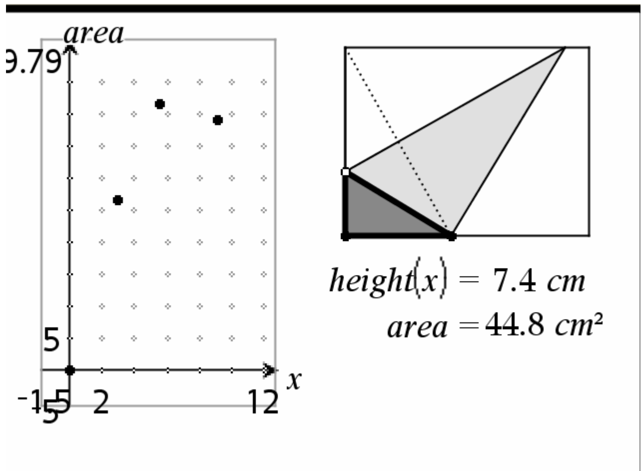
Measure, in centimeters, the height and base of your bottom left triangle. Write these measurements on your piece of paper. Be sure to note which is the height and which is the base.

Measure your triangle, and collect data from your class to enter in the spreadsheet below.

	A tri_h...	B tri_b...	C tri_area	D
◆			='tri_heigh	
1	9.1	8.6	39.13	
2	2.9	18.4	26.68	
3	5.5	15.1	41.525	

A1 | 9.1

Look at the class data on a scatterplot to determine the measurements of the triangle with the maximum area.

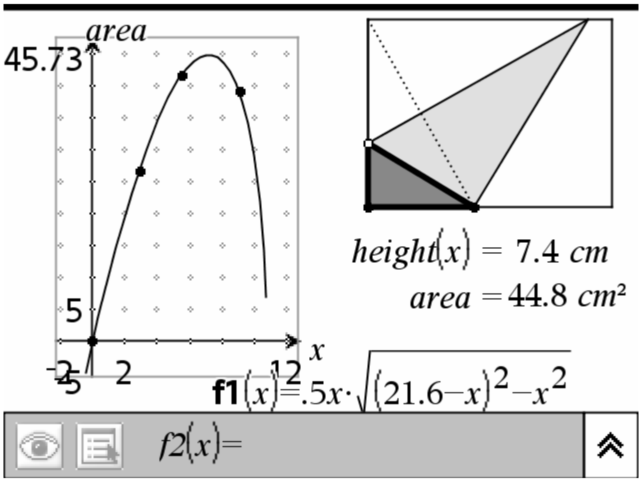


ALGEBRA I, II EXTENSION

Question

Use the Pythagorean Theorem to find an equation for the base of the triangle.

Answer



CALCULUS EXTENSION

Question

Use Calculus to prove why this fold gives the triangle with the maximum area.

Answer

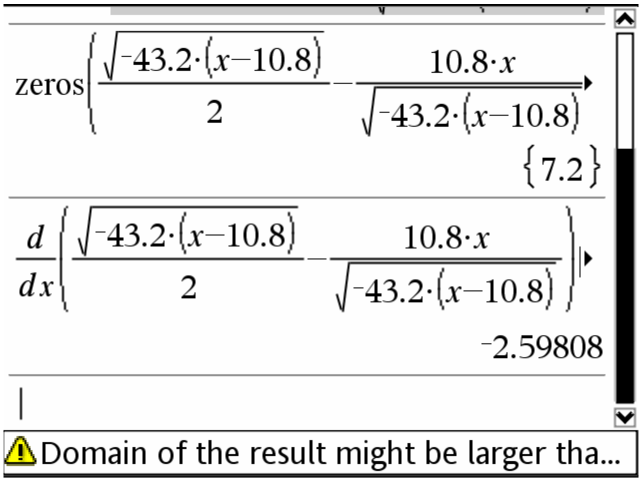
$$\frac{d}{dx}(f1(x))$$

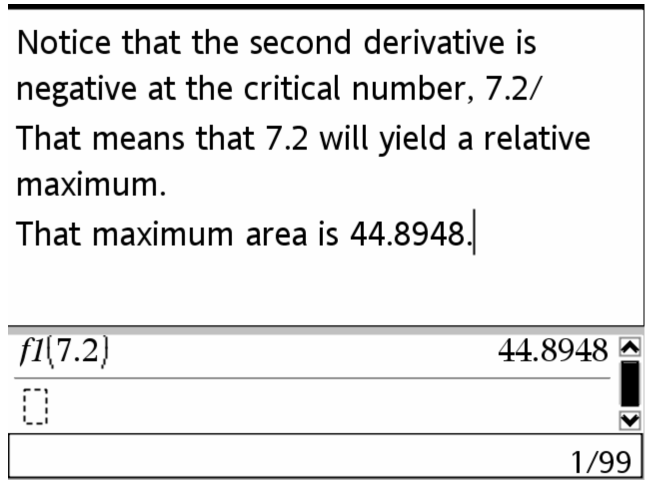
$$\frac{\sqrt{-43.2\cdot(x-10.8)}}{2}-\frac{10.8\cdot x}{\sqrt{-43.2\cdot(x-10.8)}}$$

$$\text{zeros}\left(\frac{\sqrt{-43.2\cdot(x-10.8)}}{2}-\frac{10.8\cdot x}{\sqrt{-43.2\cdot(x-10.8)}}\right.$$

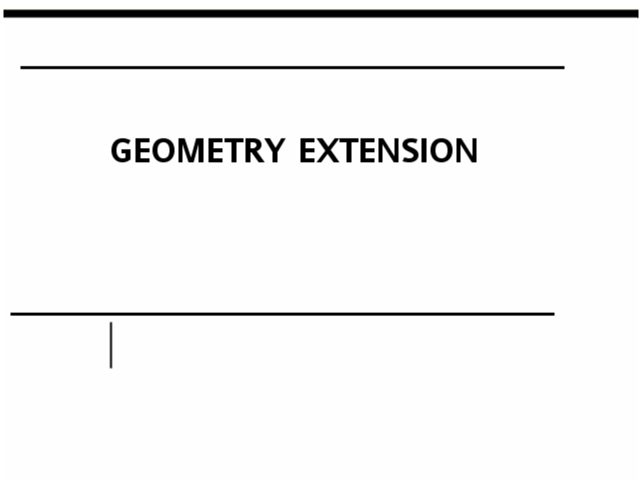
$$\{7.2\}$$

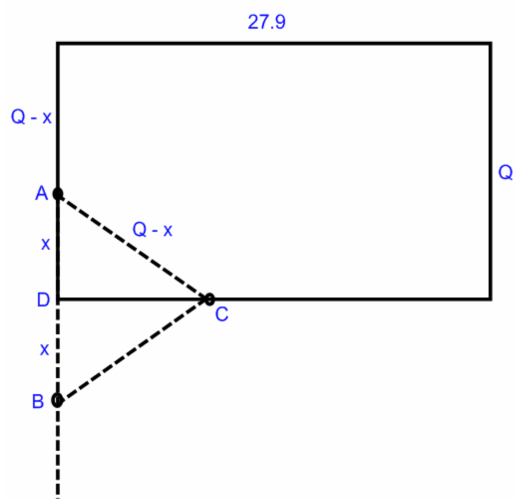
2/99





The following geometric solution was supplied by David Pollack of Youngstown State University.





Consider the figure to the right above. Let x represent the height of the right triangle ADC that we wish to have the maximum area. Let Q represent the height of the rectangle and the width of the rectangle will be 27.9 as shown.
Reflect triangle ADC over the base of the rectangle. This creates isosceles triangle ABC and the length of segment AB is 2x. The perimeter of triangle ABC = 2x + Q – x + Q – x = 2Q. So the perimeter of triangle ABC is fixed, 2Q.
We wish to maximize the area of a triangle that has a fixed perimeter. That triangle must be an equilateral triangle. That is, triangle ABC is equilateral.

Since its perimeter is 2Q, each side is $\frac{2Q}{3}$.

AB = 2x = $\frac{2Q}{3} \Rightarrow$ x = $\frac{Q}{3}$. For our example, Q = 21.6. So x = $\frac{Q}{3} = \frac{21.6}{3} = 7.2$, the same we got by several other methods. This problem was solved by graphical methods, algebraic methods, geometric methods, and utilizing CAS and calculus. All solutions coincided. ☺

MANUFACTURING A GALLON CAN
A MINIMIZATION PROBLEM

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A metal can in the shape of a rectangular solid with a square base (top and bottom) is to be manufactured at a minimum cost for materials. It is to hold one gallon of heavy–duty paint remover. The sides are to be made of one gauge of metal that costs $1.10 per square foot and the top and bottom must be made with a heavier gauge metal that costs $2.75

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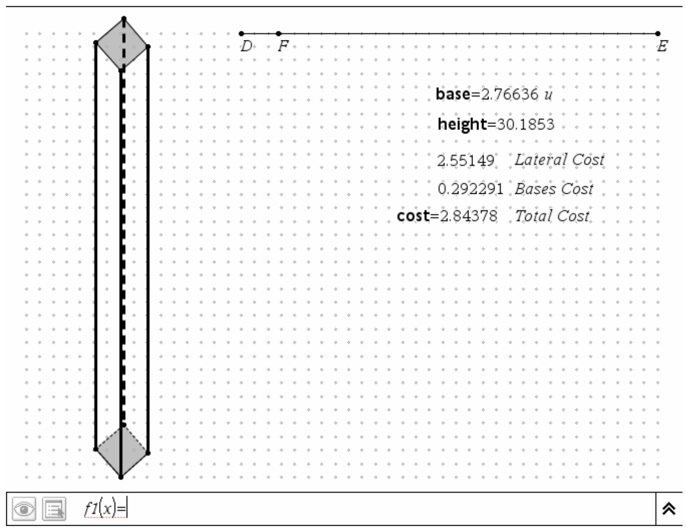
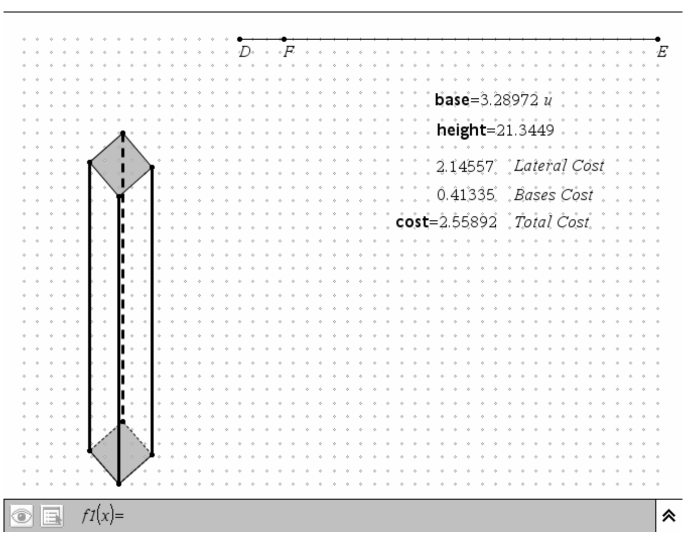
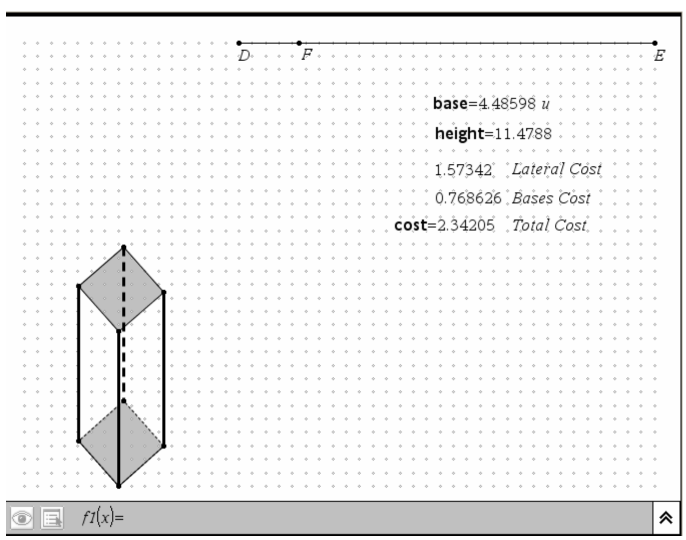
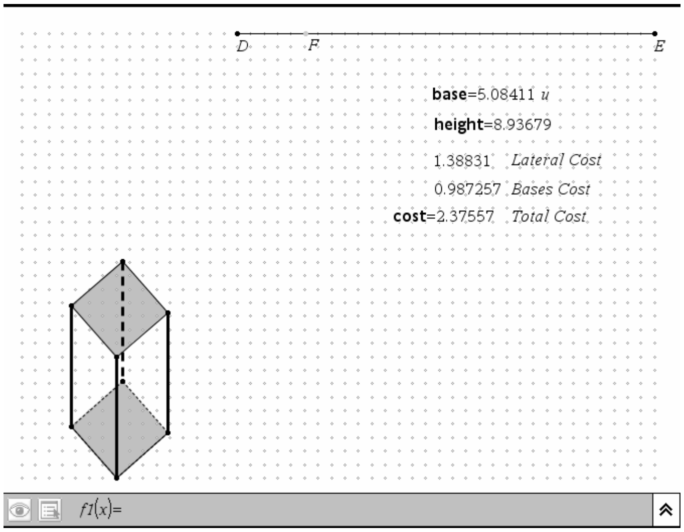
per square foot.

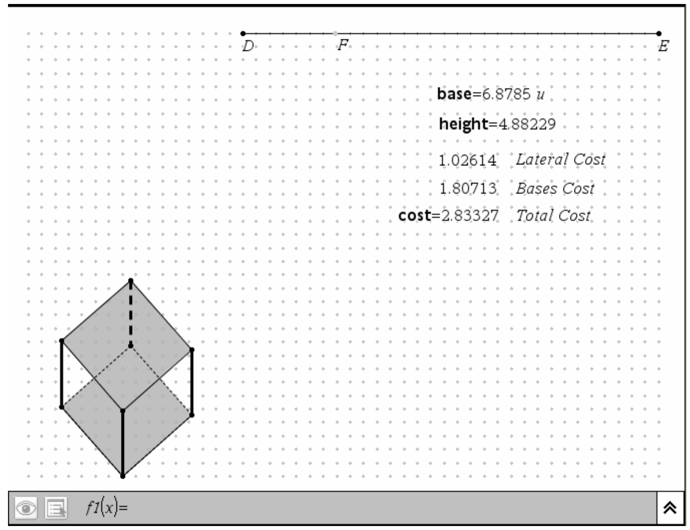
Your responsibility is to find the dimensions of the can (to the nearest hundredth of an inch) that minimizes the cost (to the nearest tenth of a penny).

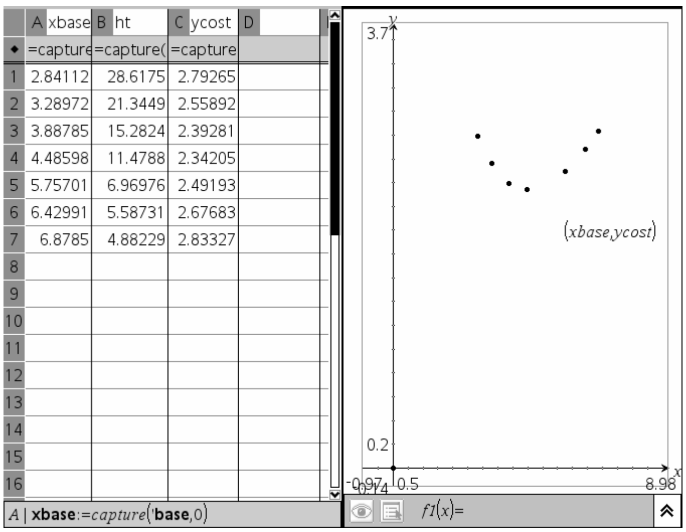




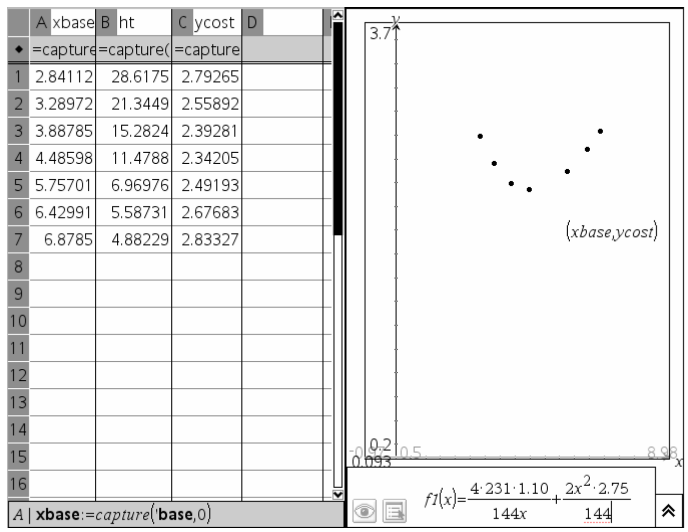








	A xbase	B ht	C ycost	D
◆	=capture	=capture(	=capture	
1	2.84112	28.6175	2.79265	
2	3.28972	21.3449	2.55892	
3	3.88785	15.2824	2.39281	
4	4.48598	11.4788	2.34205	
5	5.75701	6.96976	2.49193	
6	6.42991	5.58731	2.67683	
7	6.8785	4.88229	2.83327	



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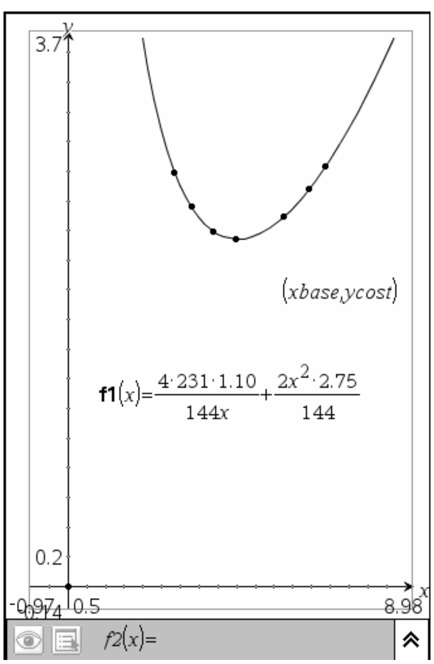

Using CAS Calculus:

Take the derivative of the Total Cost function stored in f1.
Take its derivative and find the zeros of the derivative to find the critical number(s).
Evaluate the second derivative at the critical number, 4.52089.
Since the value of the second derivative at the critical number is positive (.229167), that means that it yields a relative minimum.
The relative minimum value is 2.34191.

$\frac{d}{dx}(f1(x))$	$.076389\cdot x-\frac{7.05833}{x^2}$
$\text{zeros}\left(.076388888888888\cdot x-\frac{7.0583333333333}{x^2},x\right)$	$\{4.52089\}$
$\frac{d}{dx}\left(.076388888888888\cdot x-\frac{7.0583333333333}{x^2}\right)$	$\frac{14.1167}{x^3}+.076389$
$\frac{14.116666666667}{x^3}+.076388888888888 \mid x=4.52089$	$.229167$
$f1(4.52089)$	2.34191

Now to generalize using CAS:
With the same parameters, now find the dimensions of the can that contains g-gallons that can be made with a minimum cost. Let g be the number of gallons.

Now the solution is a function of g, the number of gallons in the can.

Notice that when g = 1, we obtain the same solution as we did before.

Define $totalcostg(x)=\frac{4\cdot x\cdot 231\cdot g\cdot 1.1}{144\cdot x^2}+\frac{2\cdot x^2\cdot 2.75}{144}$	*Done*
$\frac{d}{dx}(totalcostg(x))$	$.076389\cdot x-\frac{7.05833\cdot g}{x^2}$
$\text{zeros}\left(.076388888888888\cdot x-\frac{7.0583333333333\cdot g}{x^2},x\right)$	$\left\{4.52089\cdot g^{\frac{1}{3}}\right\}$
$\frac{d}{dx}\left(.076388888888888\cdot x-\frac{7.0583333333333\cdot g}{x^2}\right) \mid x=4.52089\cdot g^{\frac{1}{3}}$	$.229167$
$totalcostg\left(4.52089\cdot g^{\frac{1}{3}}\right)$	$2.34191\cdot g^{\frac{2}{3}}$
$2.3419058579891\cdot g^{\frac{2}{3}} \mid g=1$	2.34191
$2.3419058579891\cdot g^{\frac{2}{3}} \mid g=2$	3.71754